Simulating the static magnetic response of thin film superconducting devices

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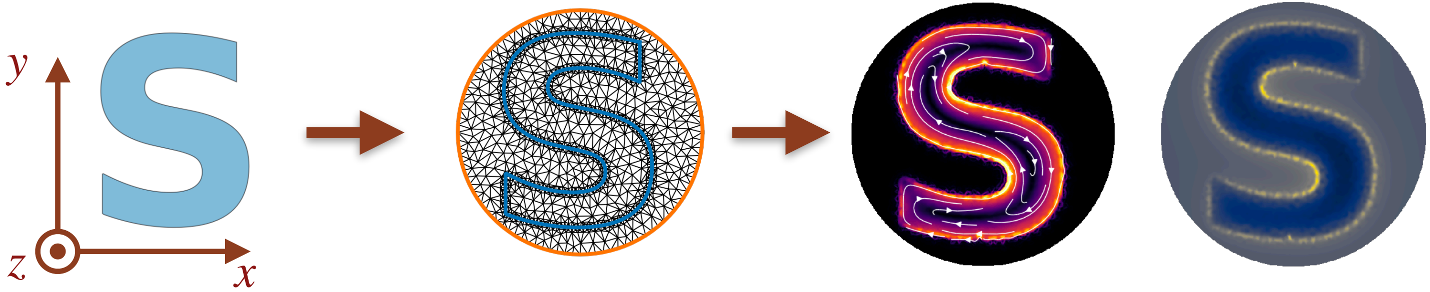
Goal



2D superconducting film, penetration depth Λ, applied field $\mu_0 H_{z,\,\mathrm{applied}}$

Numerically solve 2D London equation

Extract Meissner screening currents and self-field

Physics Wish List

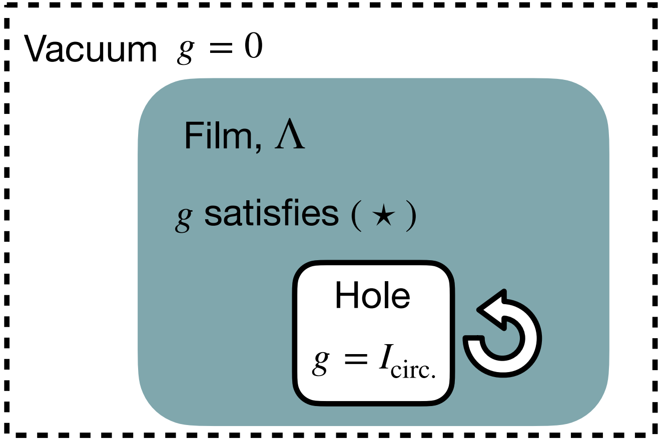
- Inhomogeneous $\Lambda(x, y)$
- Fluxoid quantization and mutual inductance in multiply-connected films
- Trapped vortices
- Stacked 2D films

Software Wish List

- User friendly
- Fast
- Open source
- Portable
- Interactive

The model

$\nabla \cdot \vec{J} = 0 \implies$ scalar stream function $g(x, y)$:
$\vec{J}(x, y) = \nabla \times (g\hat{z})$



2D London equation in terms of g:

$$\vec{H}(x, y) = \frac{\lambda^2}{d}\nabla^2 g(x, y)\hat{z} = \Lambda \nabla^2 g(x, y)\hat{z}$$

1. Brandt & Clem, PRB **69**, 184509 (2004).
2. Brandt, PRB **72**, 024529 (2005).
3. Khapaev, Supercon. Sci. Technol. (1997).
4. Kirtley, …, Supercon. Sci. Technol. (2016).

Biot-Savart in terms of g:

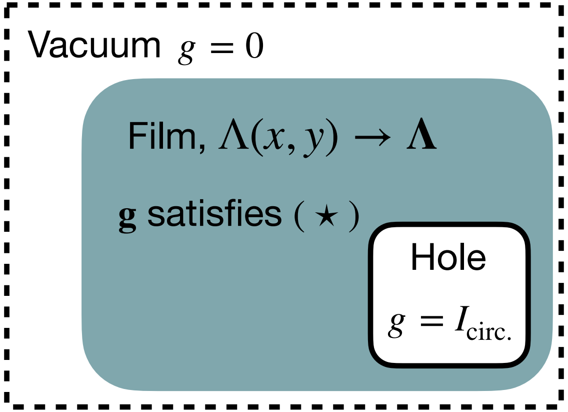
Applied field — Screening field — Total field

$$H_{z,\,\text{applied}}(\vec{r}) + \int_{\text{film}} Q_z(\vec{r}, \vec{r}')g(\vec{r}')\,\mathrm{d}^2 r' = \Lambda \nabla^2 g(\vec{r}) \qquad (\star)$$

Numerical implementation

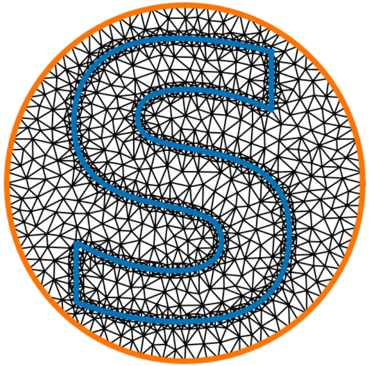
Applied field | Screening field | Total field

$$H_{z,\,\mathrm{applied}}(\vec{r}) + \int_{\mathrm{film}} Q_z(\vec{r},\vec{r}\,')g(\vec{r}\,')\,\mathrm{d}^2r' = \Lambda\,\nabla^2 g(\vec{r})$$



Discretize film and surrounding vacuum

- Delaunay triangulation —> n vertices with areas $\mathbf{w}$
- Dipole kernel Q_z —> dense $n \times n$ floating point matrix $\mathbf{Q}$
- Laplace operator ∇^2 —> sparse $n \times n$ floating point matrix $\mathbf{L}$



Solve linear system for unknown vector $\mathbf{g}$ inside the film:

$$\mathbf{h}_{z,\,\mathrm{applied}} = -\left(\mathbf{Q}\,.\,\mathbf{w}^T - \mathbf{L}\,.\,\mathbf{\Lambda}^T\right)\mathbf{g} \quad (\star)$$

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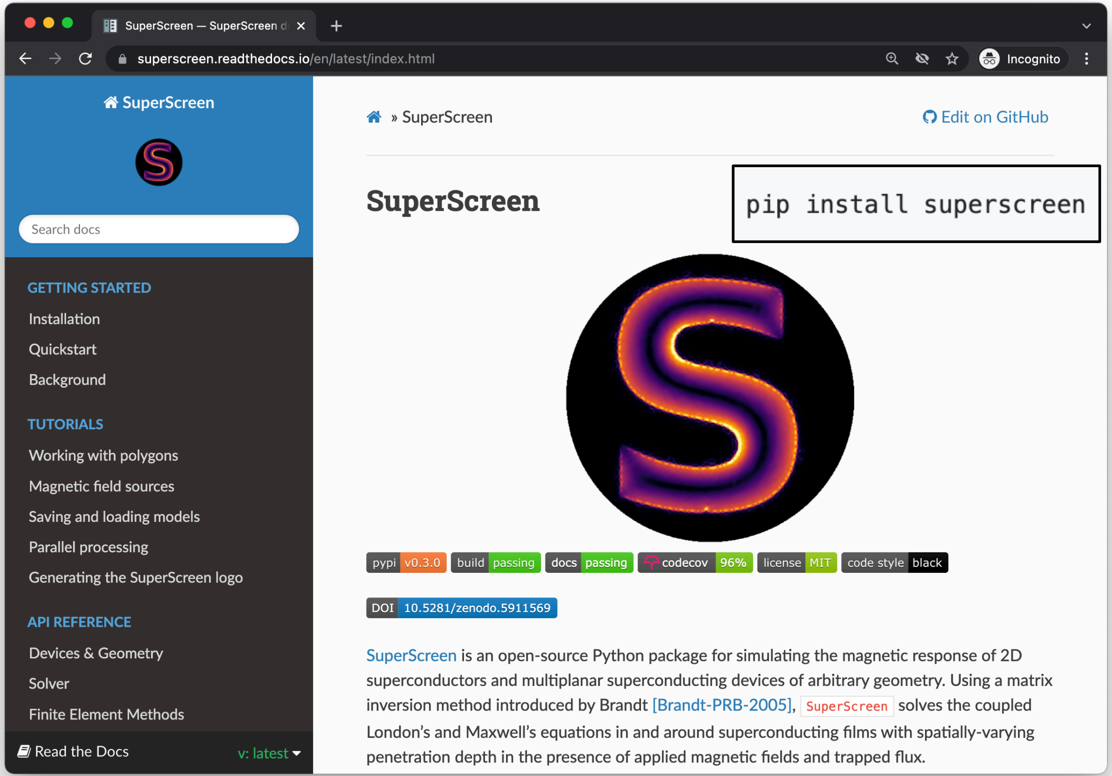
2. Kirtley, …, Supercon. Sci. Technol. (2016).

Open source software implementation



superscreen.readthedocs.io, github.com/loganbvh/superscreen

Example: Superconducting ring with a slit

```python
import superscreen as sc
from superscreen.geometry import circle, box
```

Import the package

```python
# Define the device geometry.
length_units = "um"
ro = 3  # outer radius
ri = 1  # inner radius
slit_width = 0.25
Lambda = 1  # effective penetration depth
# circle() and box() generate arrays of polygon (x, y) coordinates.
ring = circle(ro)
hole = circle(ri)
slit = box(slit_width, 1.5 * (ro - ri), center=(0, -(ro + ri) / 2))
# Define the Polygon representing the superconductor.
layer = sc.Layer("base", Lambda=Lambda)
film = sc.Polygon.from_difference(
    [ring, slit, hole], name="ring_with_slit", layer="base"
)
bounding_box = sc.Polygon("bounding_box", layer="base", points=circle(1.2 * ro))
# Create a Device and generate and plot the computational mesh.
device = sc.Device(
    film.name,
    layers=[layer],
    films=[film],
    abstract_regions=[bounding_box],
    length_units=length_units,
)
```

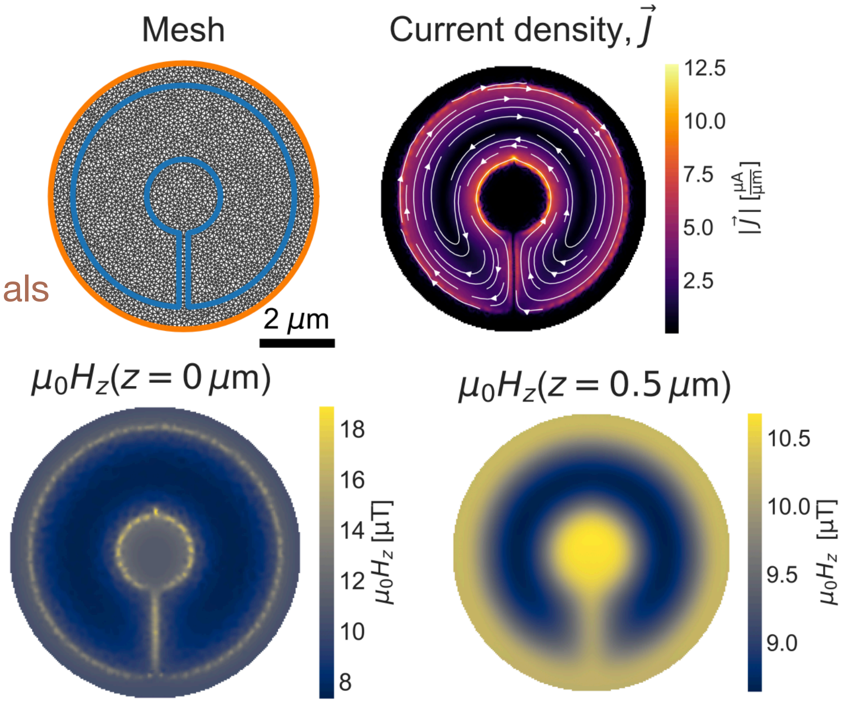
Define geometry and materials

```python
device.make_mesh(min_points=3500, optimesh_steps=None)
device.plot(mesh=True)
# Calculate the device's response to a uniform applied field.
applied_field = sc.sources.ConstantField(10)
solution = sc.solve(device, applied_field=applied_field, field_units="uT")[-1]
```

Solve the model

```python
# Visualize the solution.
# Plot the current density evaluated at each layer in the Device.
solution.plot_currents()
# Plot the magnetic field evaluated at each layer in the Device.
solution.plot_fields()
# Plot the field evaluated at any points in space.
solution.plot_field_at_positions(device.points, zs=0.5)
```

Visualize the results

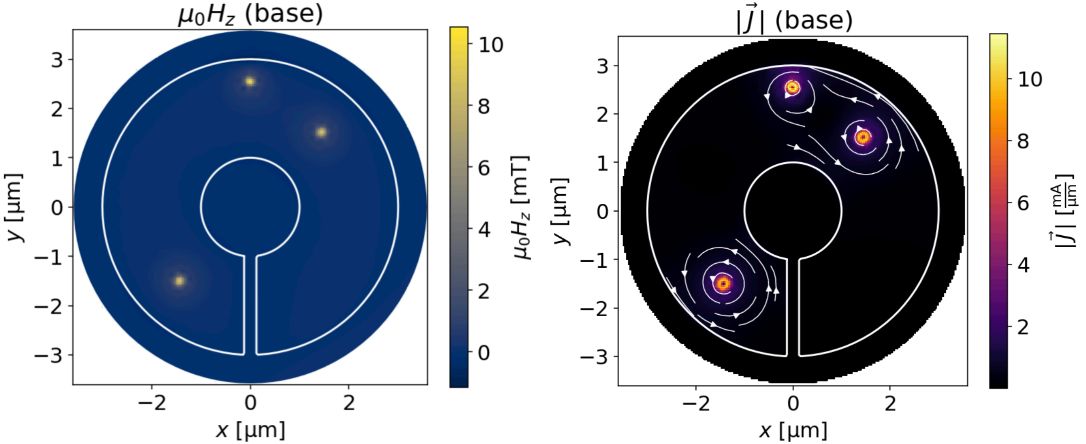


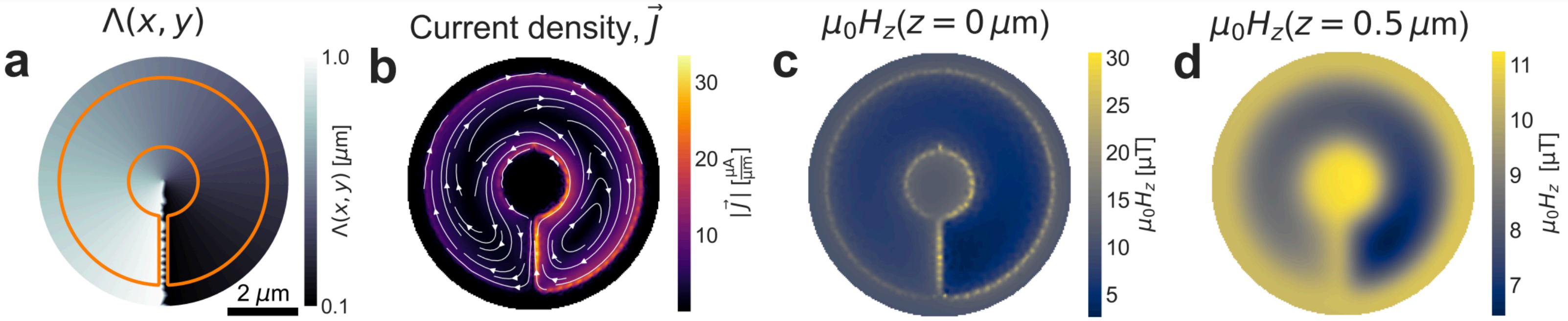
Logan Bishop-Van Horn, superscreen.readthedocs.io

Stanford University

Example: Trapped vortices, inhomogeneous $\Lambda(x, y)$

```python
vortices = [
    sc.Vortex(x=1.5, y=1.5, layer="base"),
    sc.Vortex(x=-1.5, y=-1.5, layer="base"),
    sc.Vortex(x=0, y=2.5, layer="base"),
]
```





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Example: Fluxoid states

Fluxoid quantization:

$$\Phi^f = \overbrace{\int_S \mu_0 H_z(\vec{r})\,\mathrm{d}^2r}^{\text{"flux part"}} + \overbrace{\oint_{\partial S} \mu_0 \Lambda(\vec{r})\,\vec{J}(\vec{r})\cdot \mathrm{d}\vec{r}}^{\text{"supercurrent part"}} = n\Phi_0, \quad n \in \mathbb{Z}$$

$\Lambda = 0.25\,\mu\mathrm{m}$, $\mu_0 H_{z,\,\mathrm{applied}} = 1\,\mathrm{mT}$

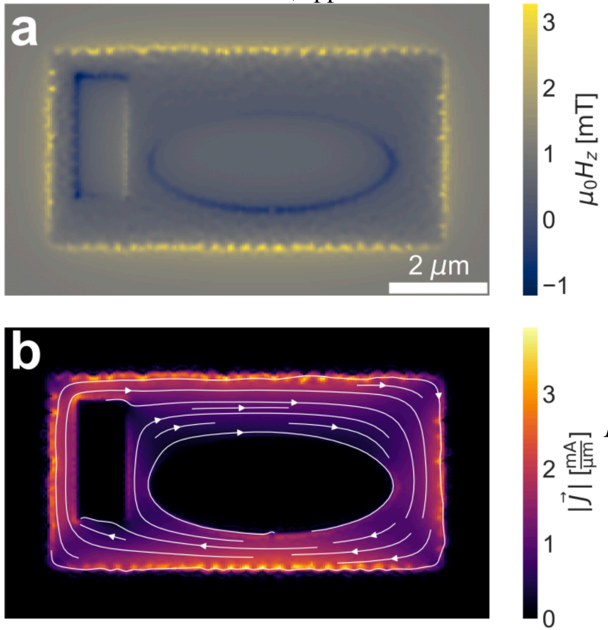
Singly-connected films, $N_h = 0$ holes

- Fluxoid quantization satisfied by solutions to 2D London equation with $n = 0$

Multiply-connected films, $N_h > 0$ holes

- Adjust circulating currents $\{I_h\}$ via gradient descent to realize desired fluxoid state $\{\Phi^f_h\}$
- Can also compute mutual inductance matrix:

$$M_{ij} = \frac{\Phi^f_{\text{hole } i}}{I_{\text{circ. } j}}$$



$\Phi^f_{\text{rectangle}} = 0\,\Phi_0$

$\Phi^f_{\text{ellipse}} = 0\,\Phi_0$

$I^{\text{rectangle}}_{\text{circ.}} = -1.07\,\mathrm{mA}$

$I^{\text{ellipse}}_{\text{circ.}} = -1.59\,\mathrm{mA}$

Model 2D superconductors

- Create complex geometries and solve for their magnetic response in a few lines of code
- Generate publication-quality visualizations
- Run on a laptop, HPC cluster, or anything in between

Share simulations with the research community

- Publish interactive Jupyter notebooks to allow others to learn from and reproduce your results

Additional Features

- Built-in magnetic field sources: distribution of dipoles, monopoles, Pearl vortices, 2D current distribution
- Robustly save/load results to/from disk
- Extensive online documentation
- Automated unit test suite

Limitations

- 2D only: $\lambda_{\mathrm{London}} > d$
- Only circulating currents, no “terminal currents”
- Slow convergence + memory-intensive for structures with many layers

Use Cases

- Inductance extraction for 2D superconducting (e.g. VdW) devices
- Modeling of magnetic microscopy, including scanning SQUID magnetometry + susceptometry

Future Work

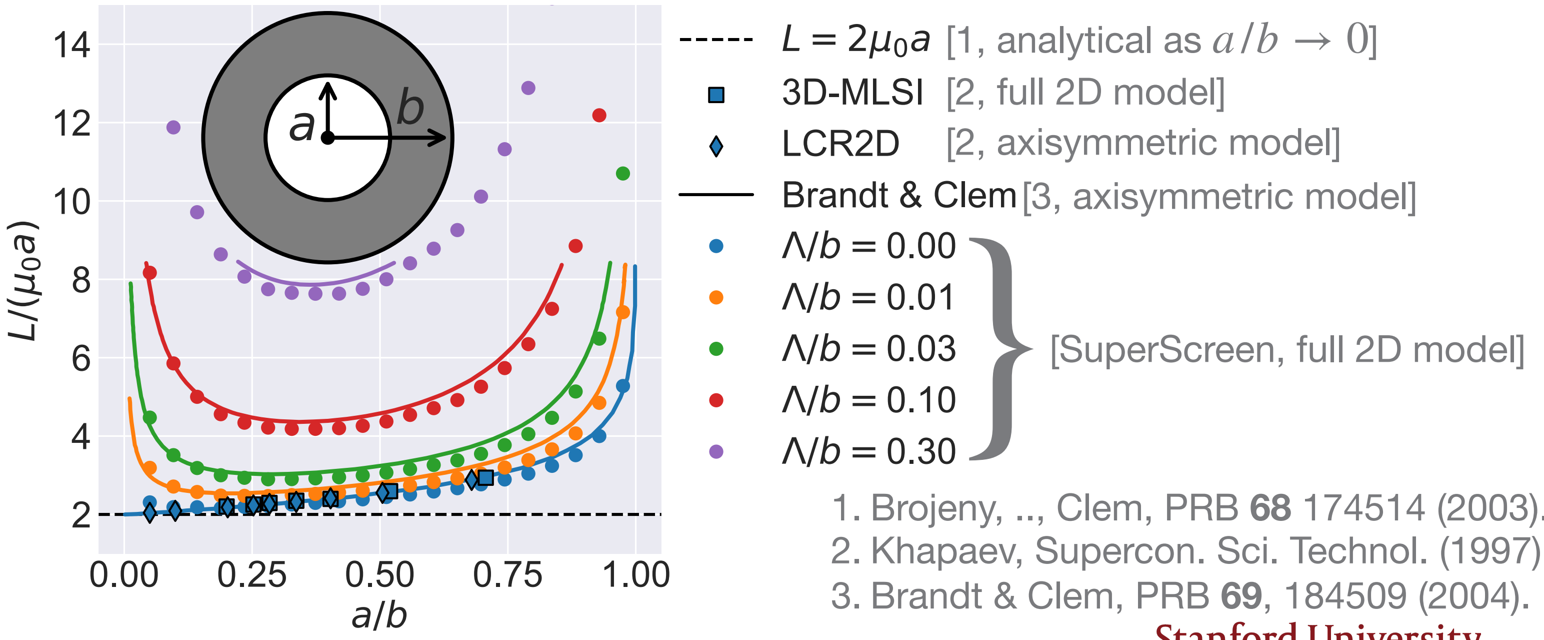
- GPU-acceleration
- Automated or adaptive mesh refinement
- Integration with CAD software/file formats

Acknowledgements

- John Kirtley: MATLAB implementation for modeling scanning SQUID microscopy:
  - Supercond. Sci. Technol. **29** (2016) 124001.
  - Rev. Sci. Instrum. 87, 093702 (2016).
- John Kirtley, Yusuke Iguchi: useful comments, discussions

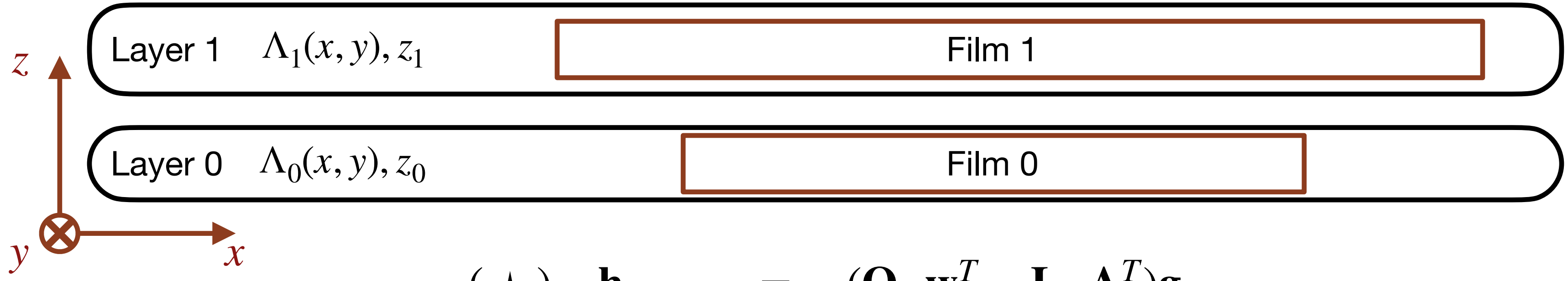
Example: Self-inductance of a flat ring

$$L = \frac{\Phi^f}{I_{\mathrm{circ.}}} = \frac{\overbrace{\int_S \mu_0 H_z(\vec{r})\,\mathrm{d}^2r}^{\text{"flux part"}} + \overbrace{\oint_{\partial S} \mu_0 \Lambda(\vec{r})\,\vec{J}(\vec{r})\cdot\mathrm{d}\vec{r}}^{\text{"supercurrent part"}}}{I_{\mathrm{circ.}}} = L_{\mathrm{geo.}} + L_{\mathrm{kin.}}$$



1. Brojeny, .., Clem, PRB **68** 174514 (2003).
2. Khapaev, Supercon. Sci. Technol. (1997).
3. Brandt & Clem, PRB **69**, 184509 (2004).

Numerical implementation: Multiple layers

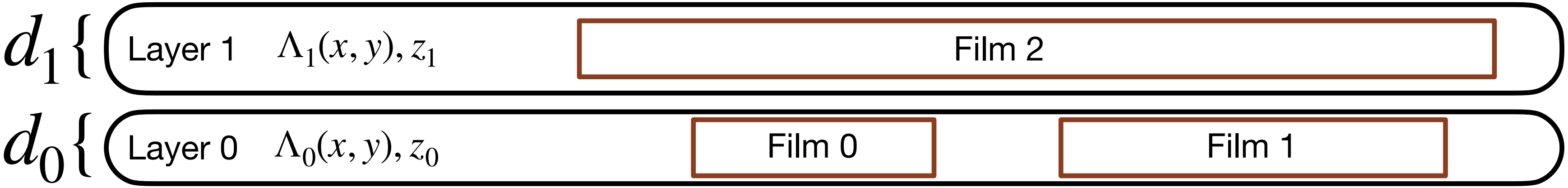


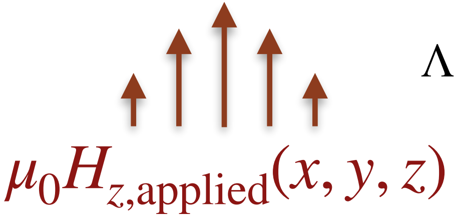
$$(\star) \quad \mathbf{h}_{z,\,\mathrm{applied}} = -(\mathbf{Q}\,.\,\mathbf{w}^T - \mathbf{L}\,.\,\mathbf{\Lambda}^T)\mathbf{g}$$

- Solve ($\star$) for each layer ℓ to obtain stream function $\mathbf{g}_\ell$
- For each layer ℓ, add to $\mathbf{h}_{z,\,\mathrm{applied}}$ the field due to $\mathbf{g}_k$ for all layers $k \neq \ell$
- Re-solve ($\star$) with updated applied field
- Repeat until solution converges

1. Brandt, PRB **72**, 024529 (2005).
2. Kirtley, …, Supercon. Sci. Technol. (2016).

Problem statement





$$\Lambda = \frac{\lambda_{\mathrm{London}}^2}{d} = \frac{\Lambda_{\mathrm{Pearl}}}{2}$$

Inputs

- $\Lambda_\ell(x, y)$, z_ℓ for each **layer** ℓ
- $x - y$ geometry for each **film** in each layer ℓ
- Applied field, $\mu_0 H_{z,\,\mathrm{applied}}(x, y, z)$

Outputs

- Sheet current density $\vec{J}_\ell(x, y)$ in each layer ℓ
- Total magnetic field $\mu_0 \vec{H}(x, y, z)$ anywhere in space

Assumptions

- Layers are 2D, $\lambda_{\mathrm{London}} > d$, and obey the London equation $\nabla \times \vec{J} = -\vec{H}/\Lambda$