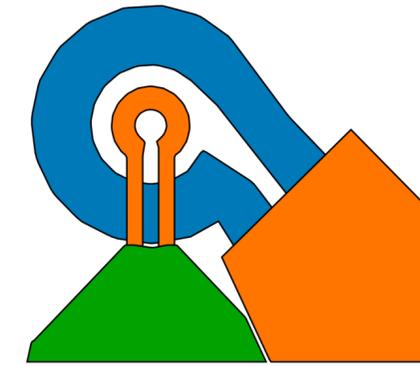
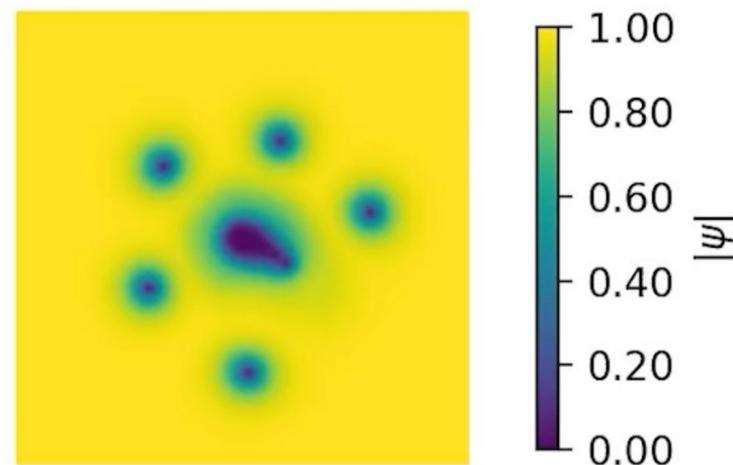


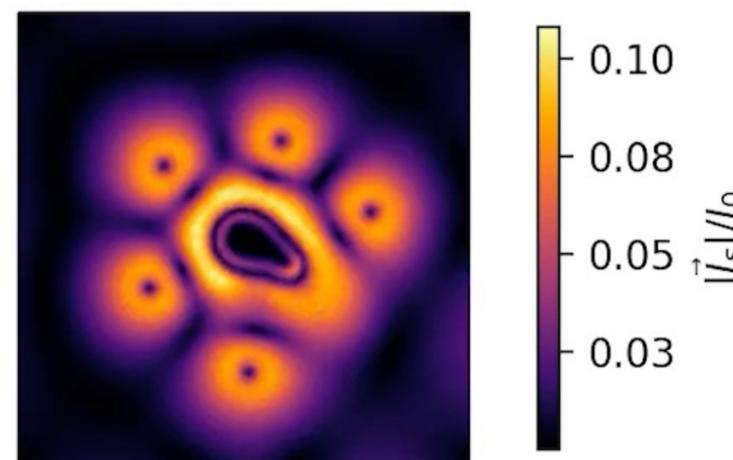
Vortex dynamics induced by scanning SQUID susceptometry



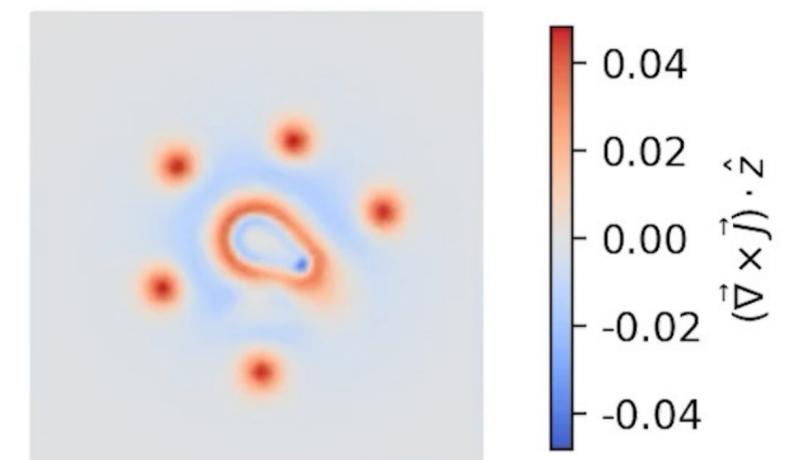
Order parameter



Supercurrent density



Vorticity



Logan Bishop-Van Horn, Eli Mueller, & Kam Moler

Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA
Department of Physics, Stanford University, Stanford, CA 94305, USA

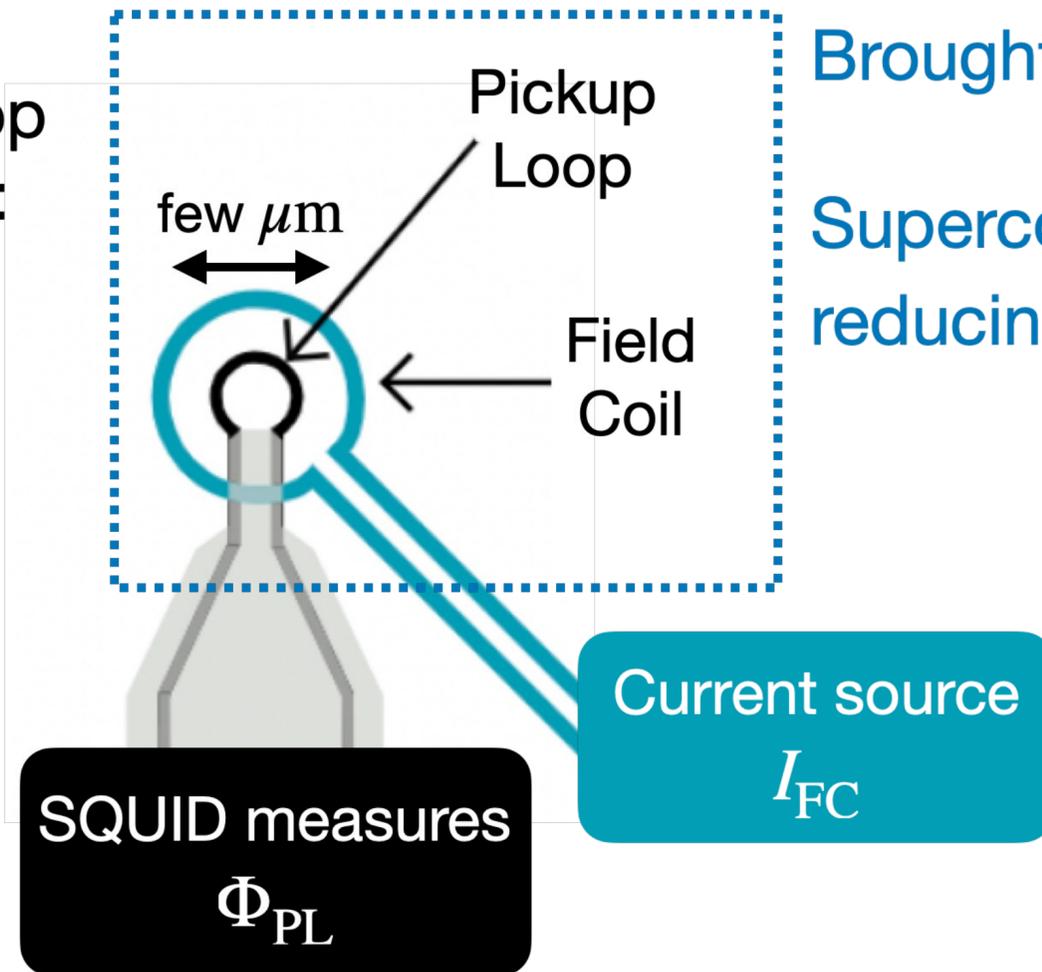
APS March Meeting 2023, Las Vegas, NV, Session K27, 2023.03.07



Scanning SQUID susceptometers measure the **local magnetic response** of superconductors

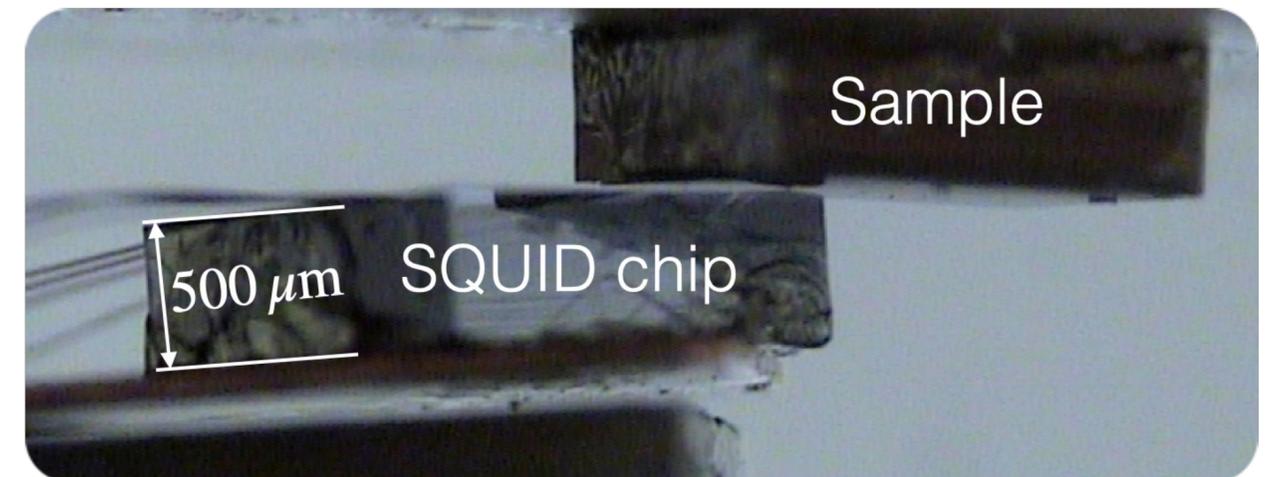
Field coil-pickup loop mutual inductance:

$$M = \Phi_{\text{PL}} / I_{\text{FC}}$$



Brought close to sample surface.

Superconducting sample screens applied field, reducing M .



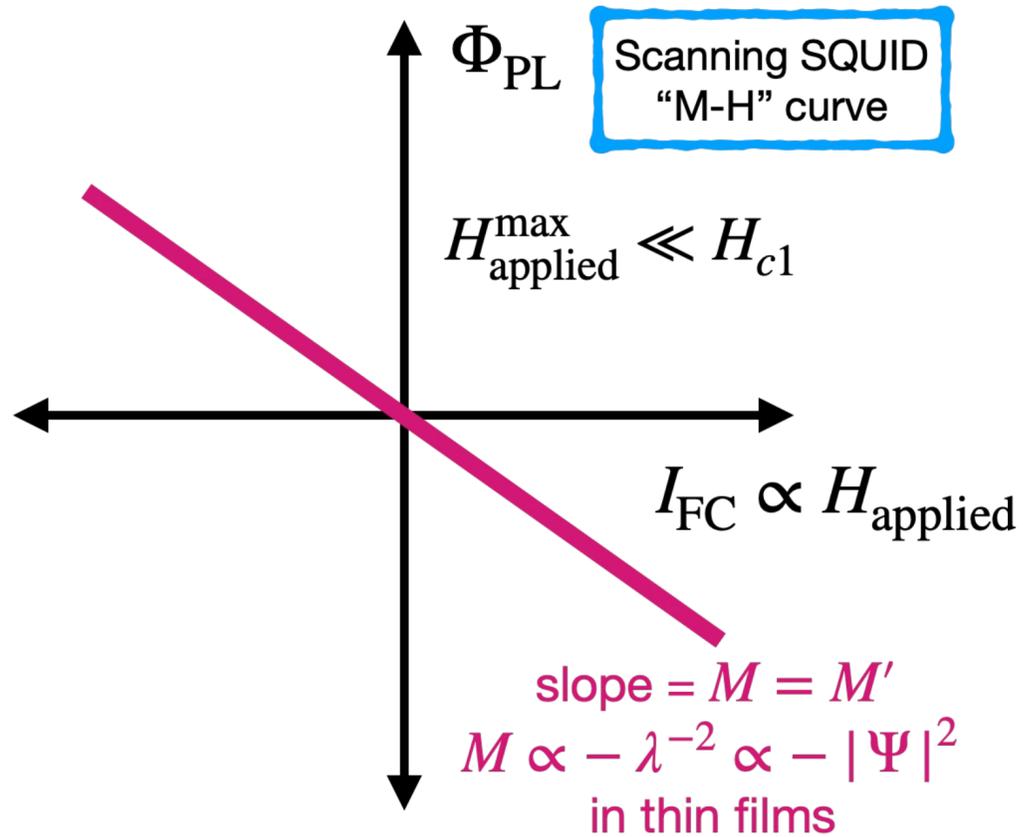
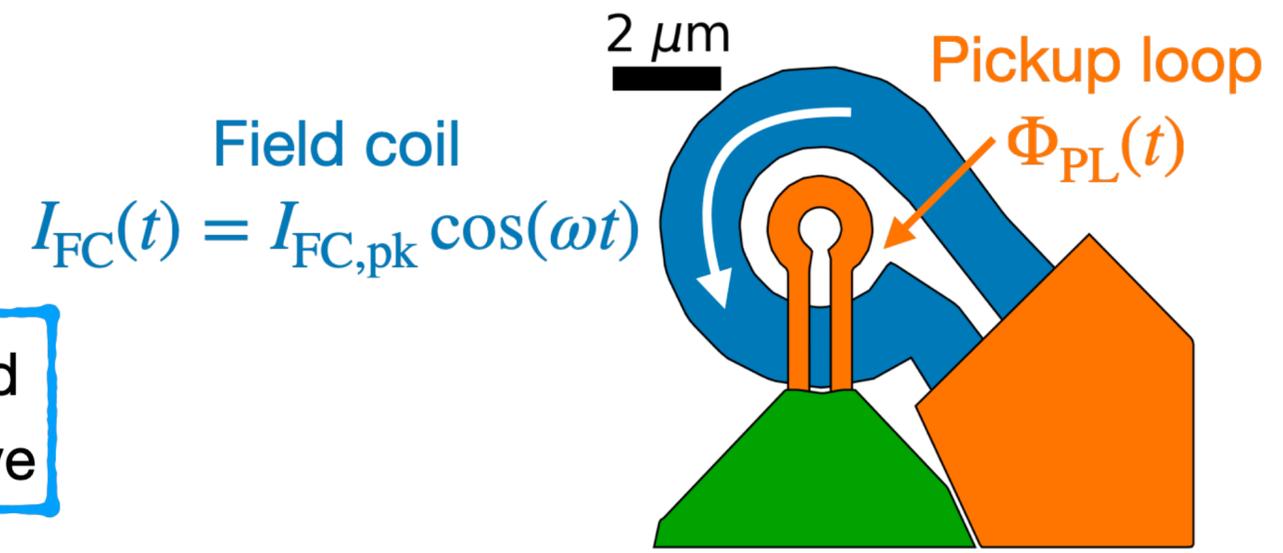
Rev. Sci. Instrum. **72**, 2361 (2001)
Rev. Sci. Instrum. **79**, 053704 (2008)
Phys. Rev. B **85**, 224518 (2012)
Rev. Sci. Instrum. **87**, 093702 (2016)

Low frequency AC magnetic response

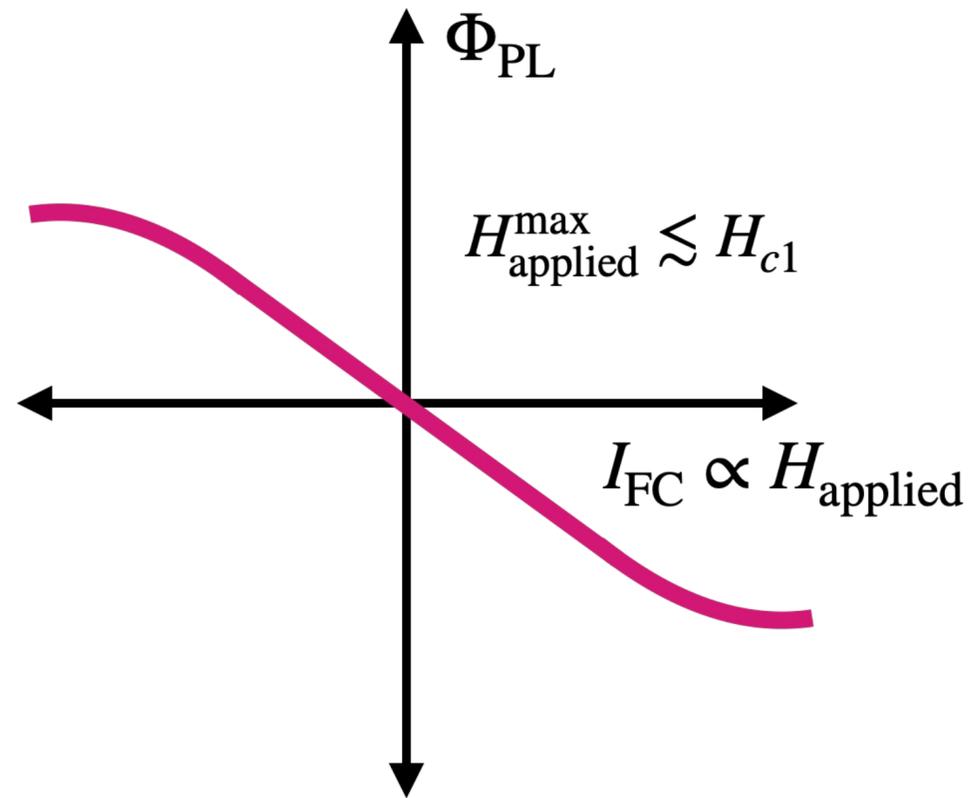
Demodulated magnetic response

$$M = \frac{\sqrt{2}}{I_{FC,pk}} \int \Phi_{PL}(t) e^{-i\omega t} dt = M' + iM''$$

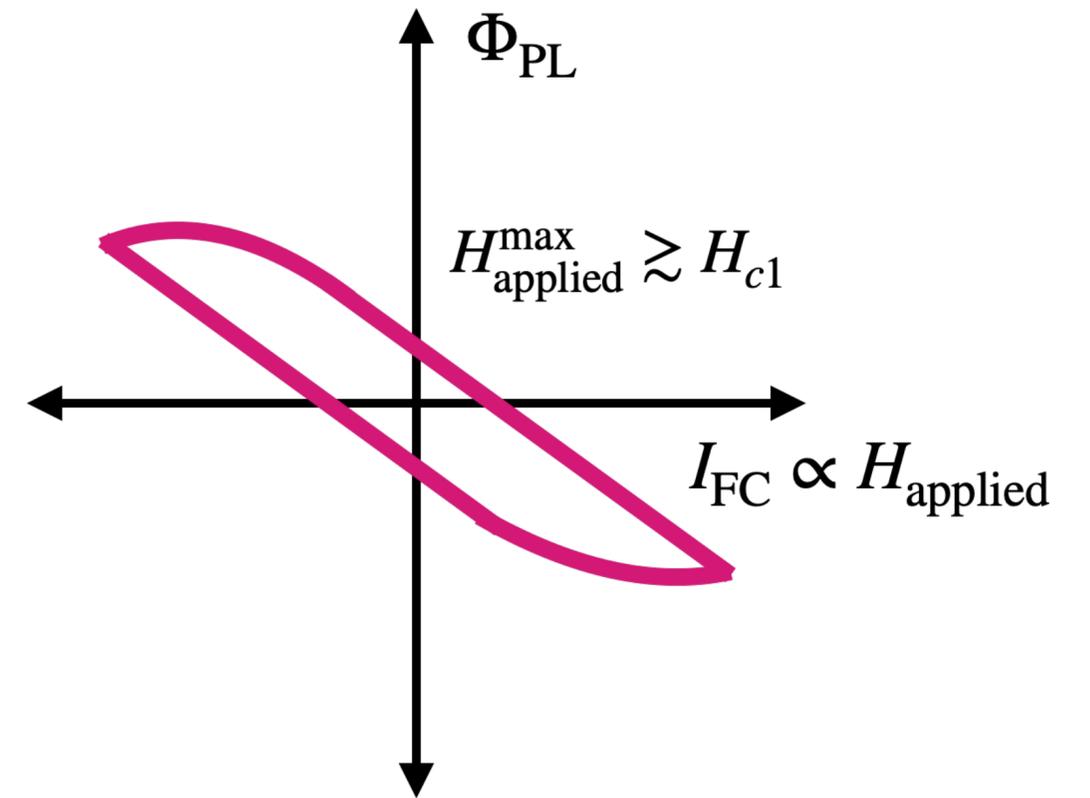
$M' = \text{Re}(M) \rightarrow$ superfluid
 $M'' = \text{Im}(M) \rightarrow$ dissipative



Linear, non-hysteretic
 (London)
 $M'' = 0$



Non-linear, non-hysteretic
 (suppressed $|\Psi|^2$)
 $M'' = 0$

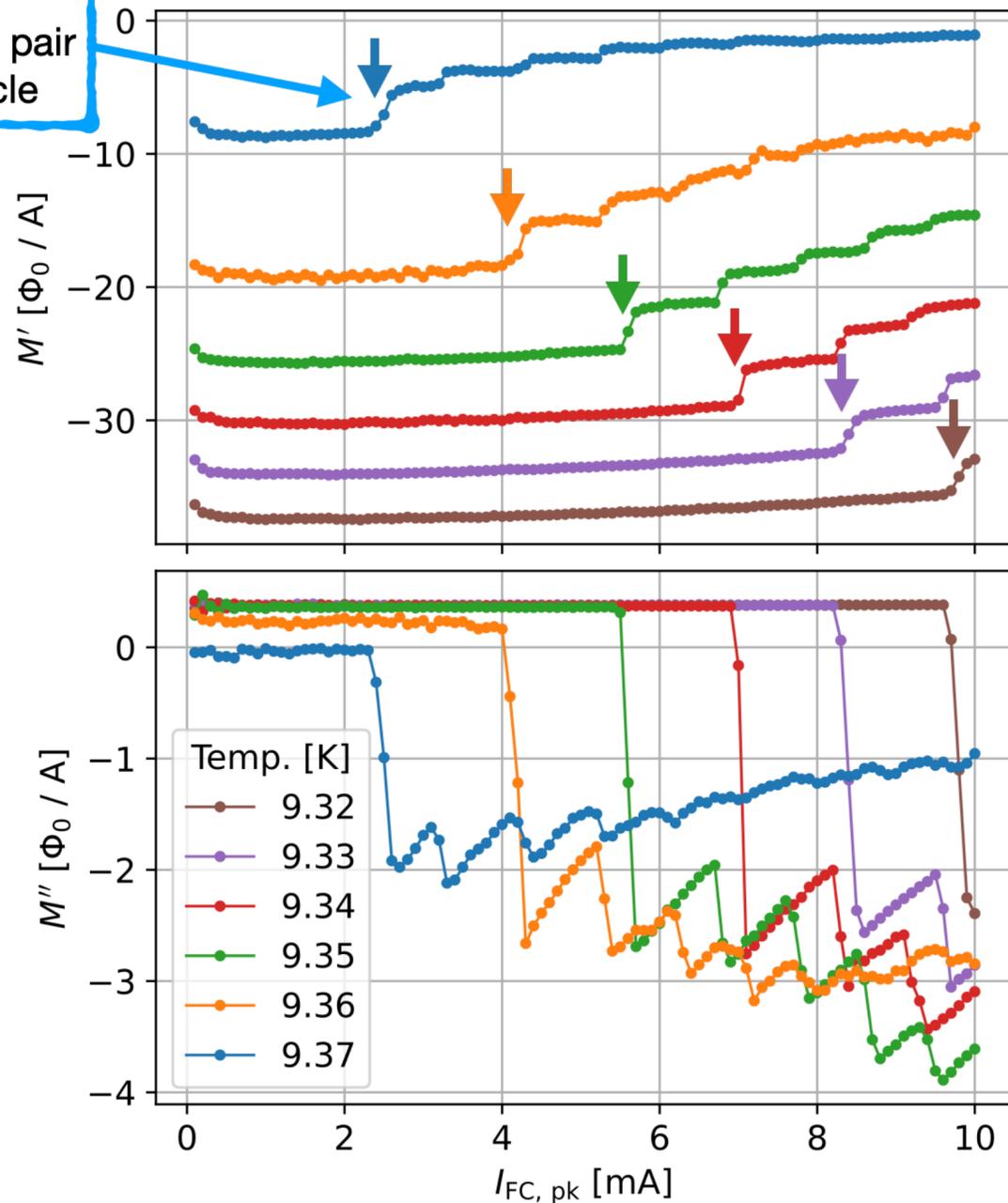


Non-linear, hysteretic
 (vortex motion)
 $M'' \neq 0$

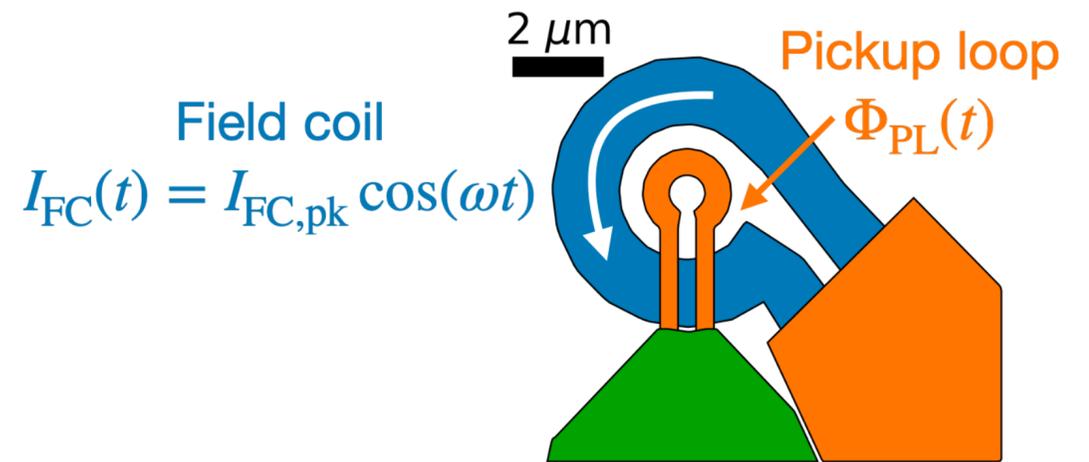
M encodes “fingerprints” of few-vortex dynamics in niobium thin film near T_c

M vs. $I_{FC,pk}$

1 induced vortex-antivortex pair per half AC cycle



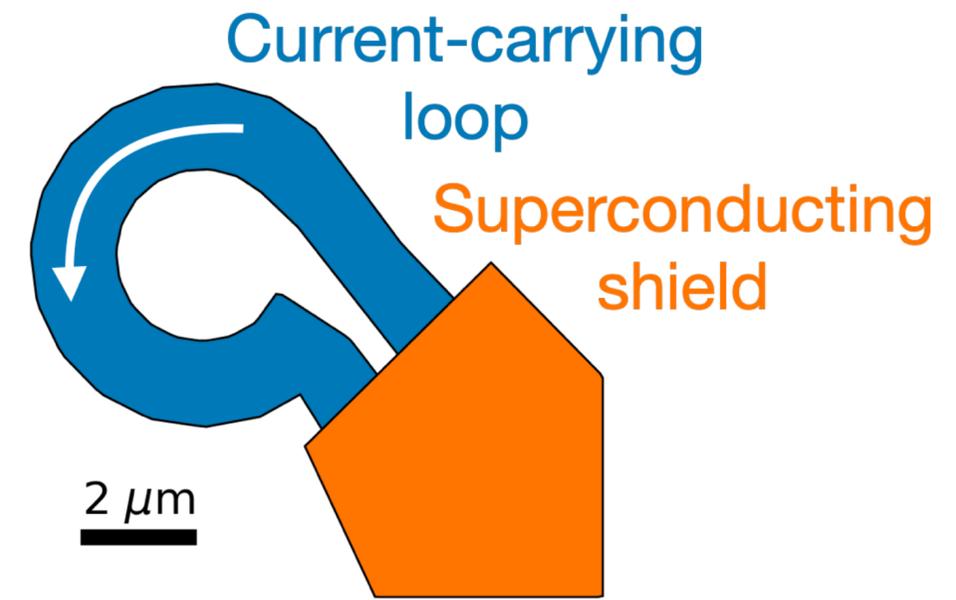
$M' = \text{Re}(M) \rightarrow$ superfluid
 $M'' = \text{Im}(M) \rightarrow$ dissipative



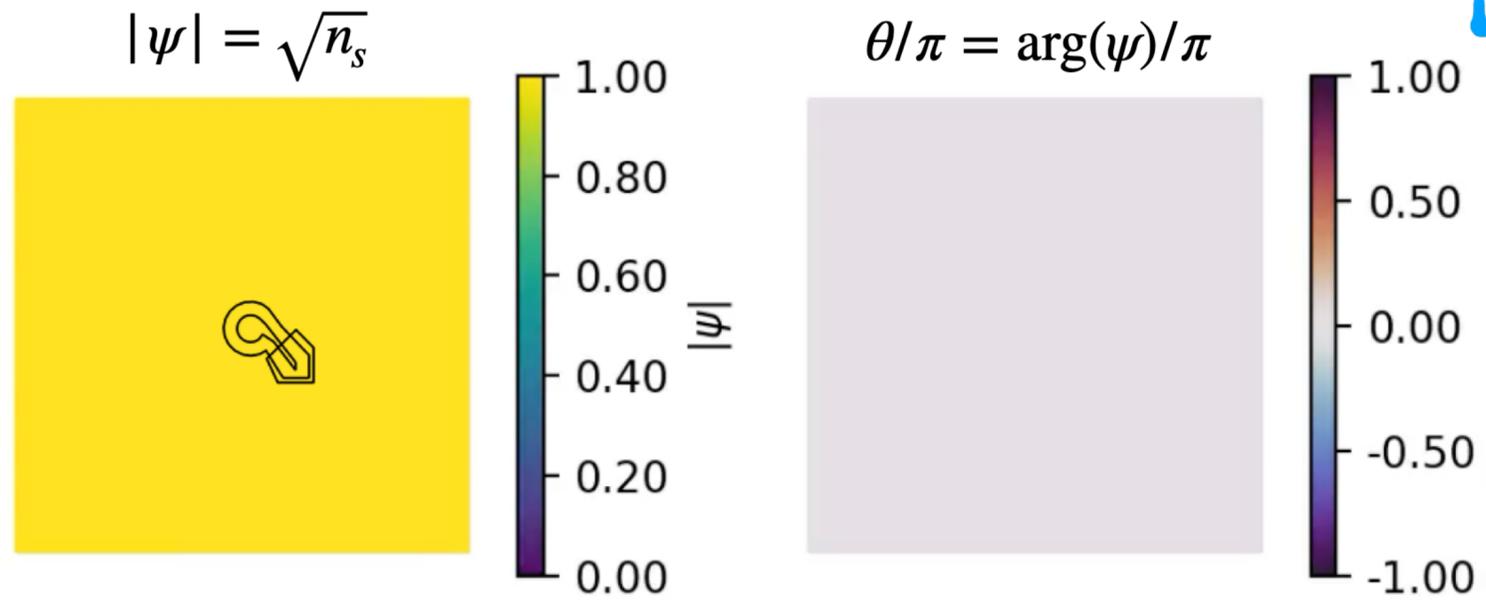
Increasing temperature,
 increasing $\xi(T)$ and $\lambda(T)$
 Increasing applied local AC field

Measurements: Eli Mueller

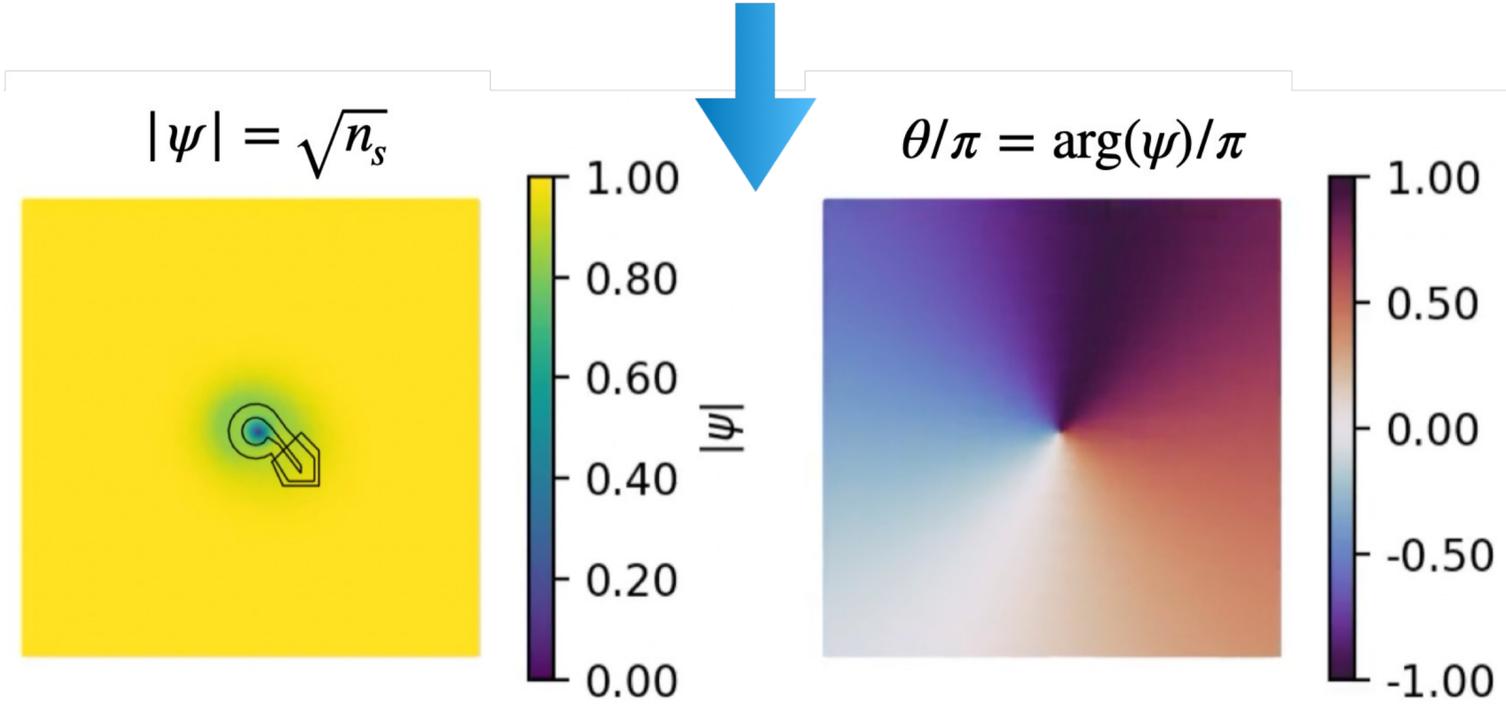
How to generate vortices with a local magnetic source?



$H_{\text{applied at edge}} \approx 0$



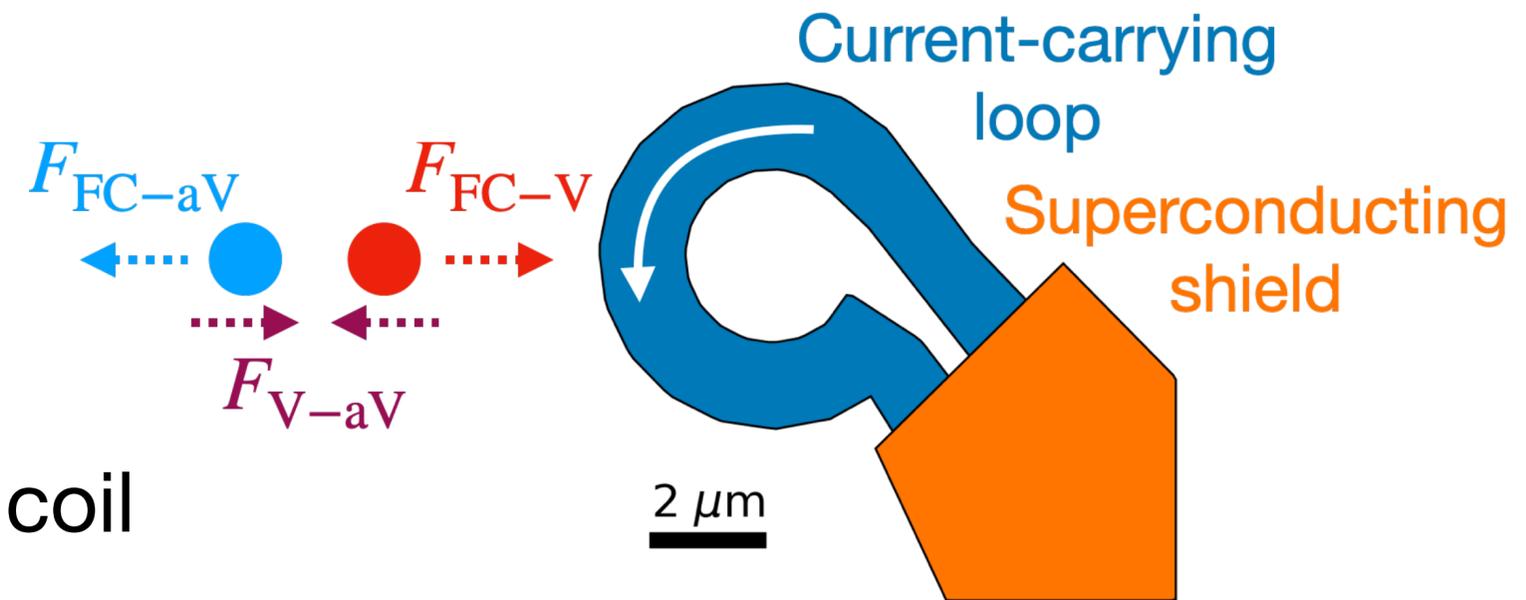
$\oint_{\text{edge}} \nabla \theta \cdot d\mathbf{r} = 0$



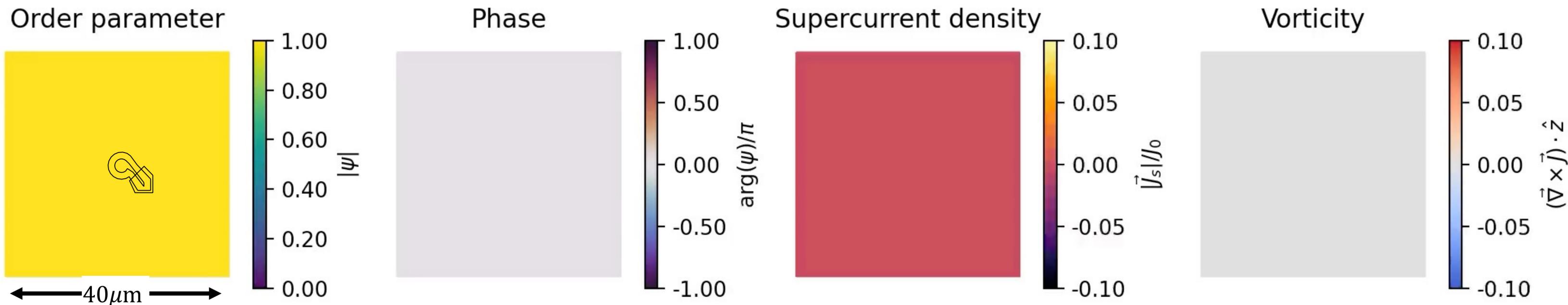
$\oint_{\text{edge}} \nabla \theta \cdot d\mathbf{r} = 2\pi$

SQUID field coil can generate a single vortex-antivortex pair in a thin film

- Vortex pulled toward center of field coil, antivortex pushed away from center of field coil
- Antivortex exits the film or becomes pinned/annihilated far from the SQUID



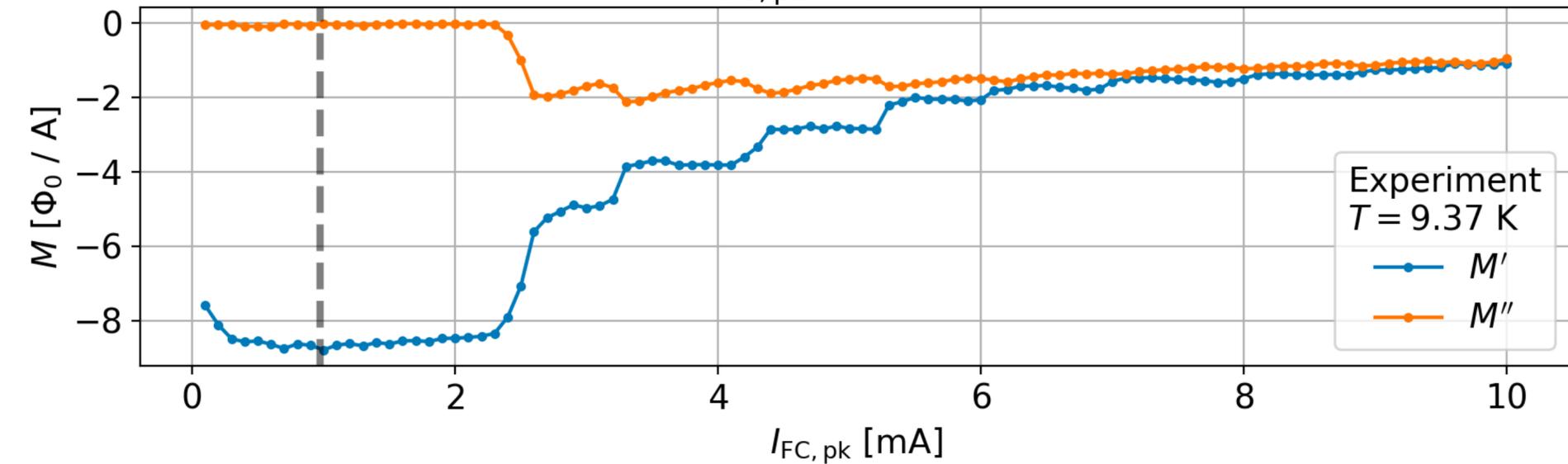
This process is analogous to a phase slip in a 1D superconducting wire.



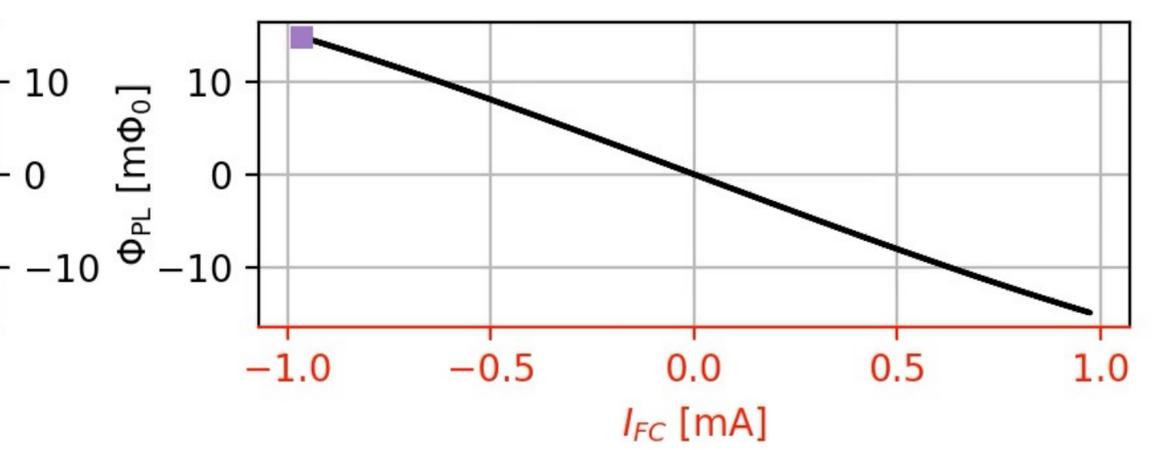
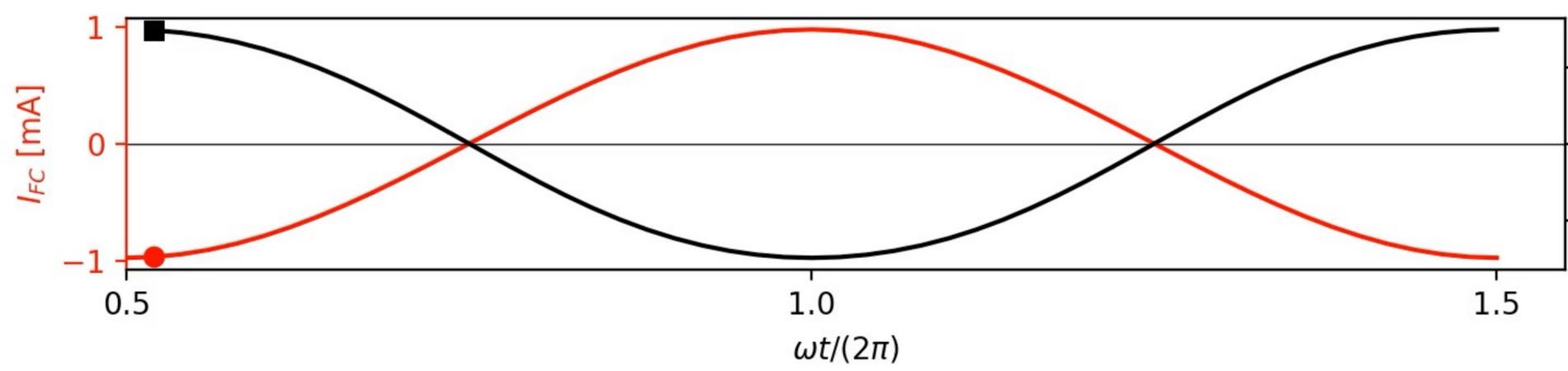
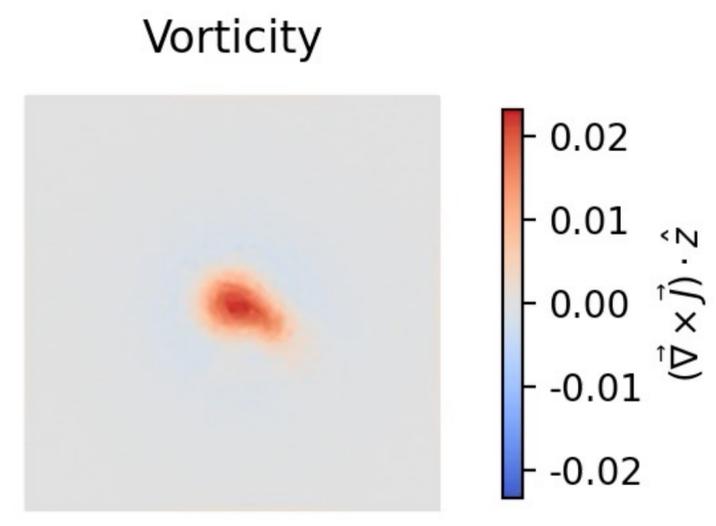
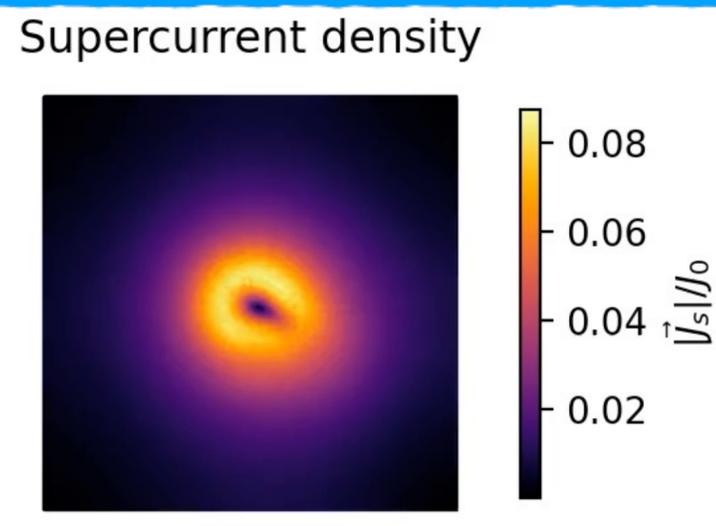
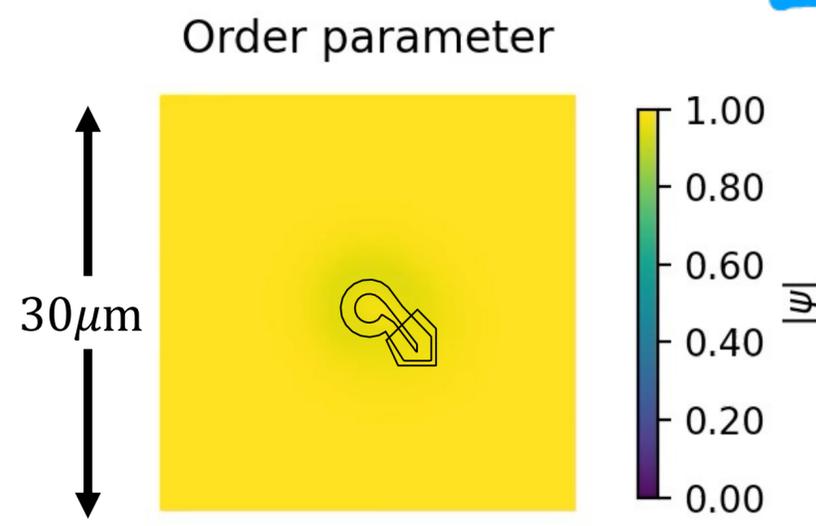
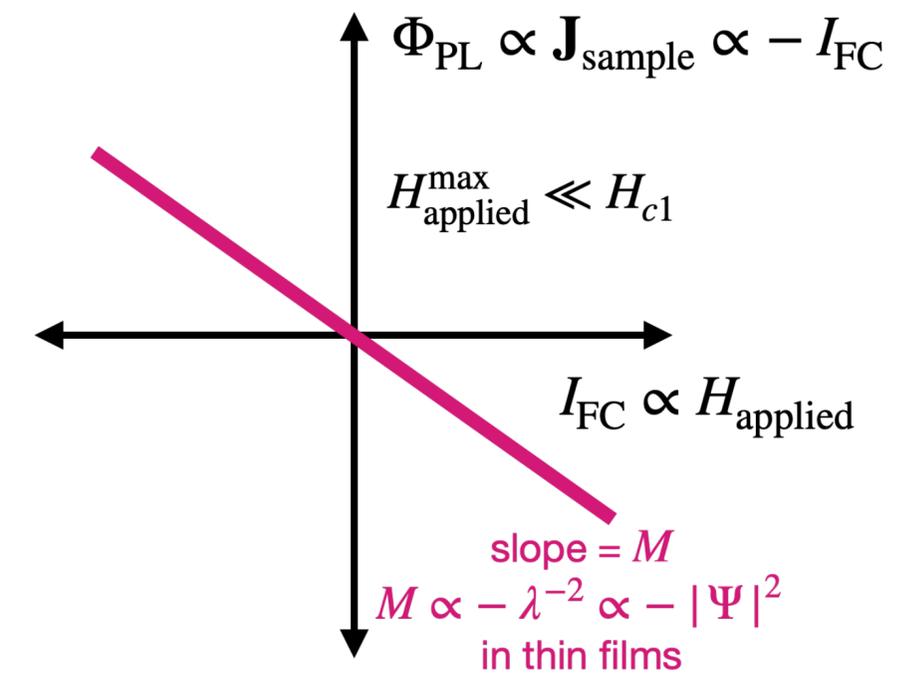
2D time-dependent Ginzburg-Landau (TDGL) simulation, $\xi = 0.9\mu\text{m}$, $\lambda = 1.35\mu\text{m}$, $I_{FC} = 2.5\text{mA}$

Simulation method: [arXiv:2302.03812](https://arxiv.org/abs/2302.03812) (2023) [and references therein]; py-tdgl.readthedocs.io

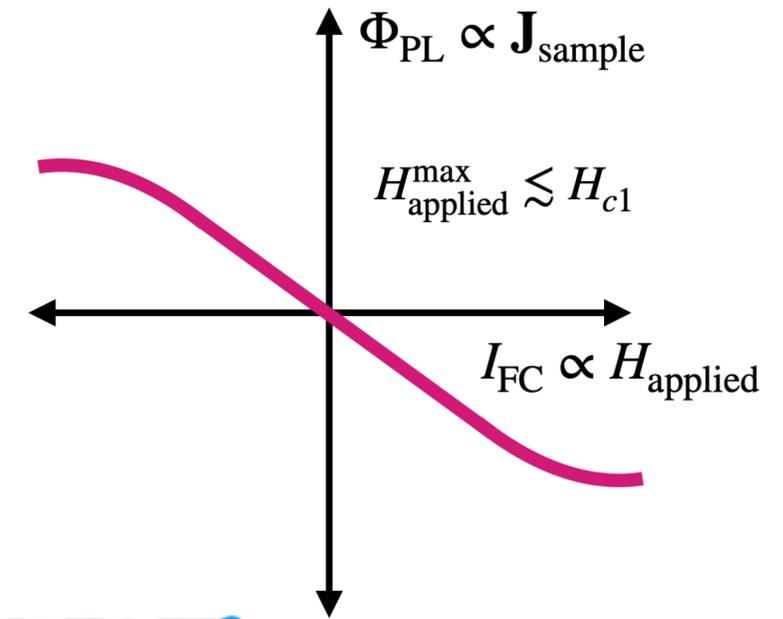
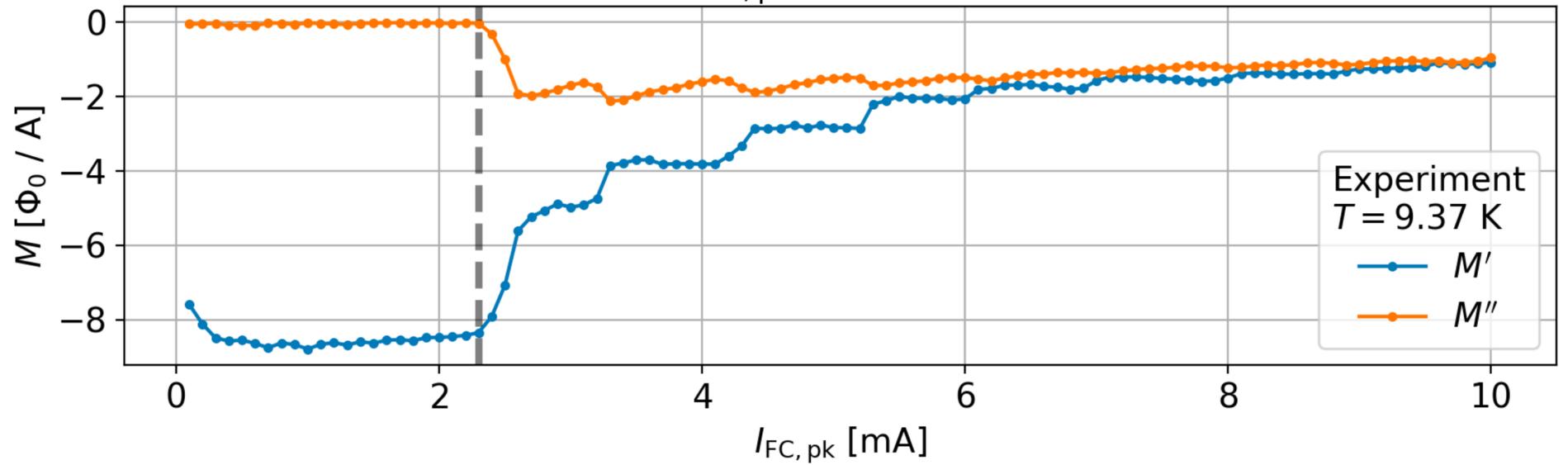
$I_{FC, pk} = 1.0 \text{ mA}$



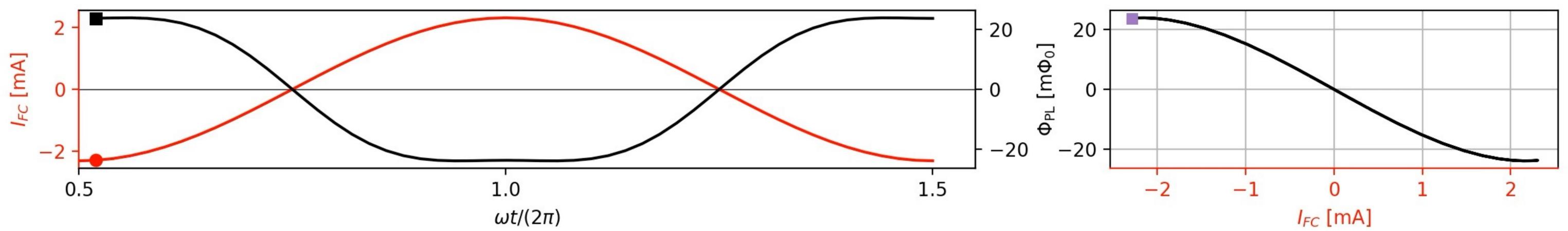
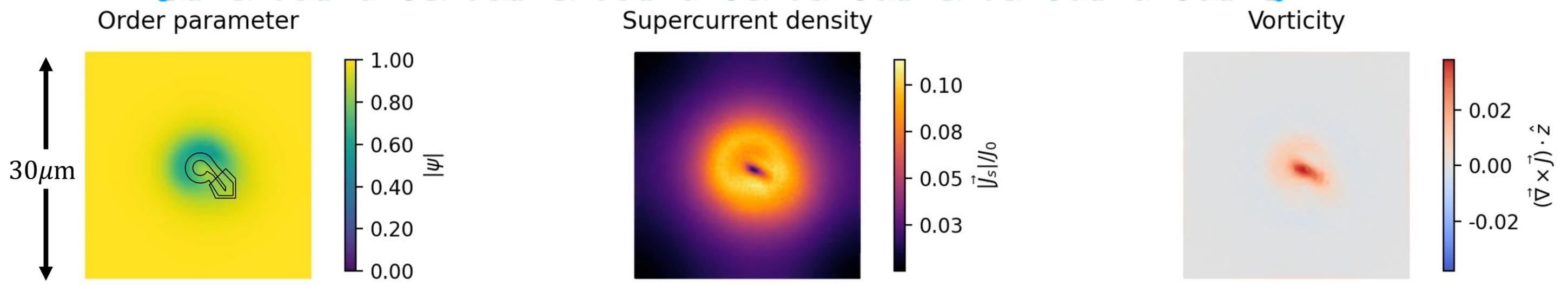
Linear, non-hysteretic diamagnetic response.



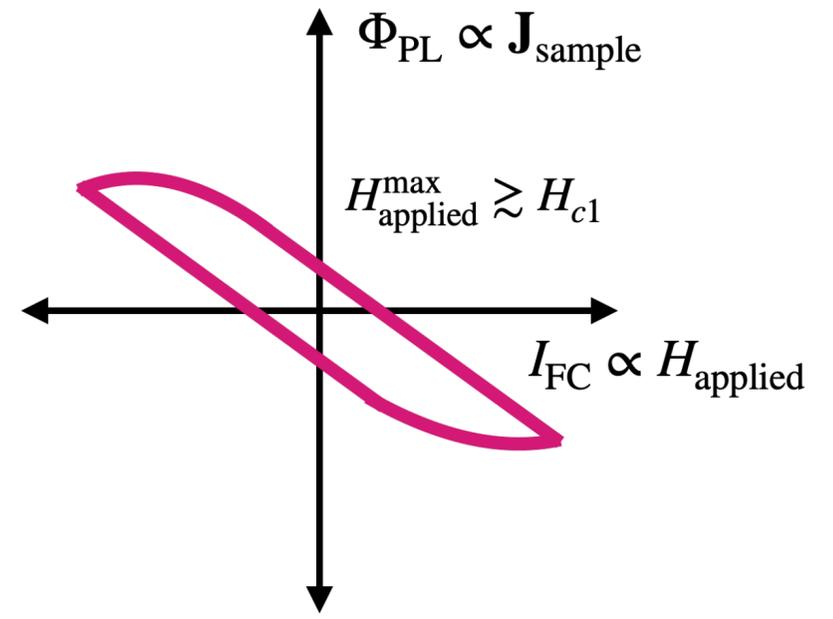
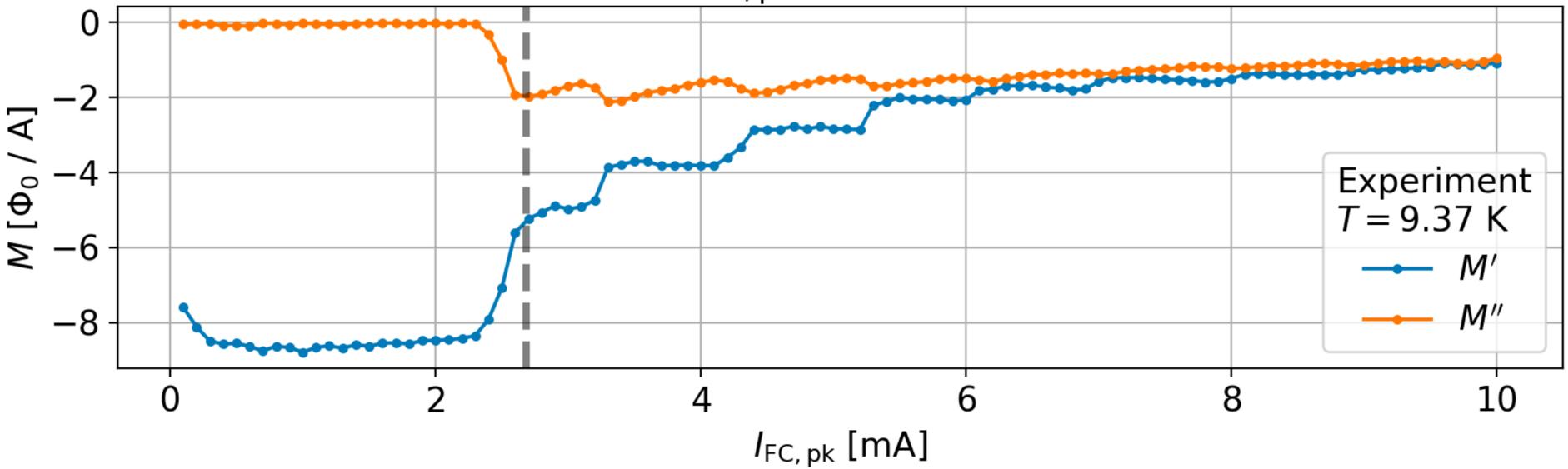
$I_{FC, pk} = 2.3 \text{ mA}$



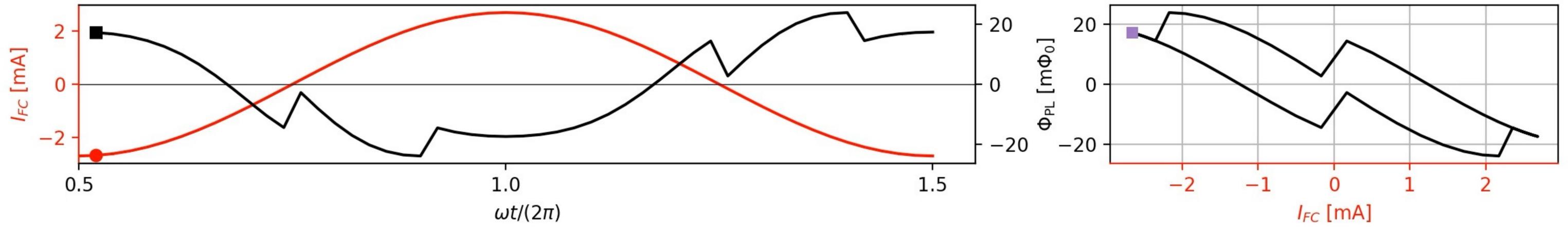
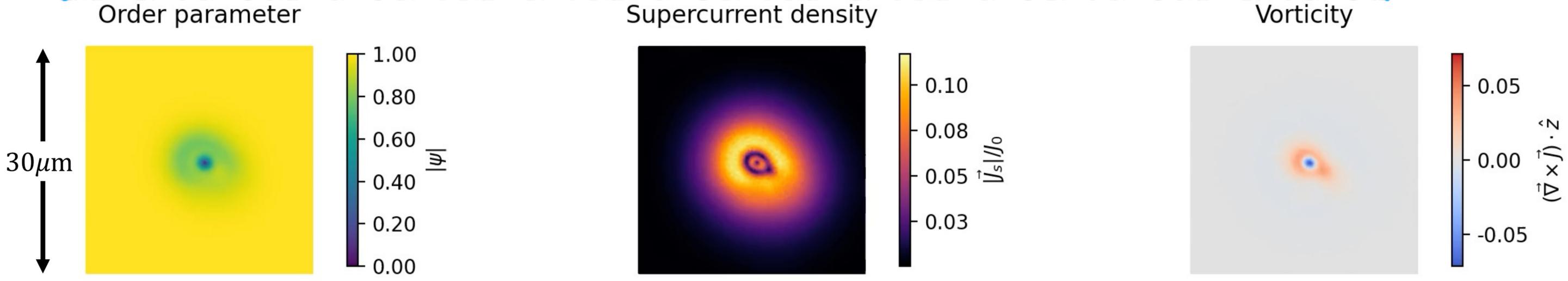
Non-linear, non-hysteretic diamagnetic response due to suppressed superfluid density.

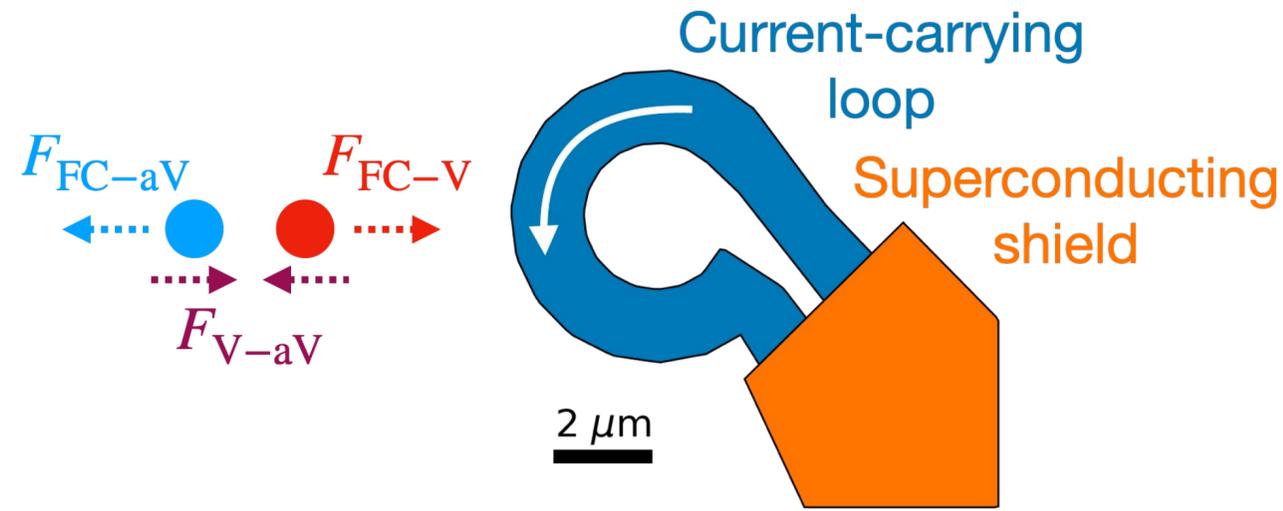
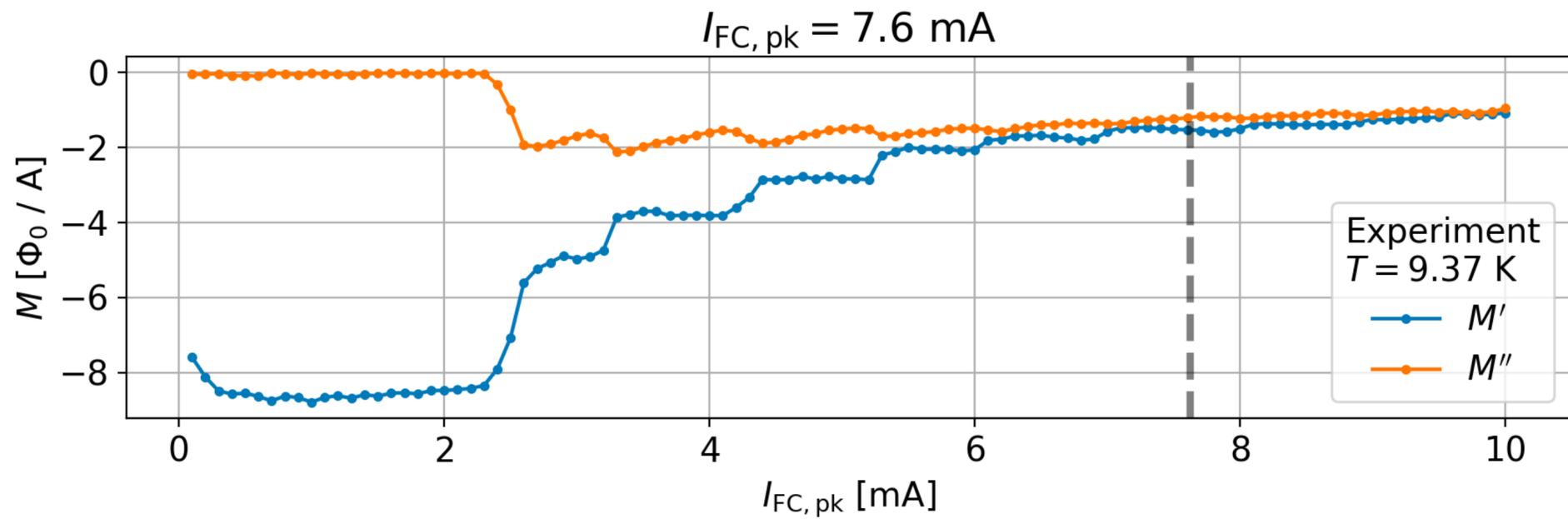


$I_{FC, pk} = 2.7 \text{ mA}$

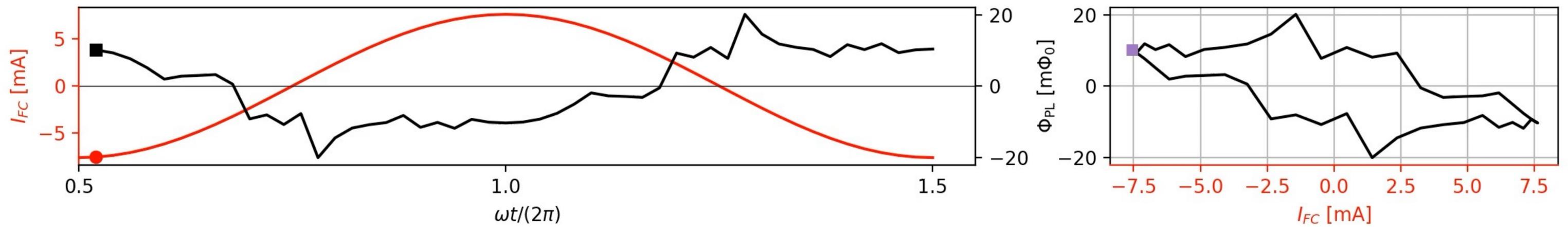
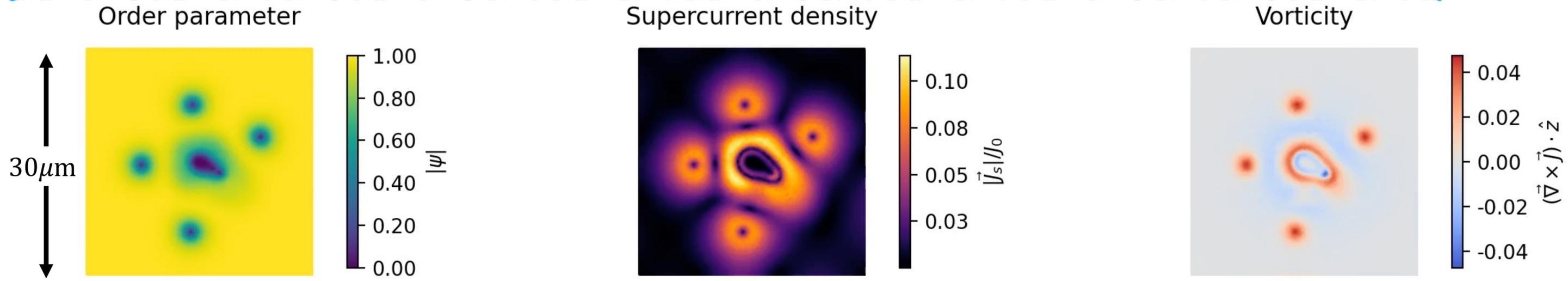


Non-linear, hysteretic response. One V-aV pair induced each half AC cycle; aV pinned far from SQUID.



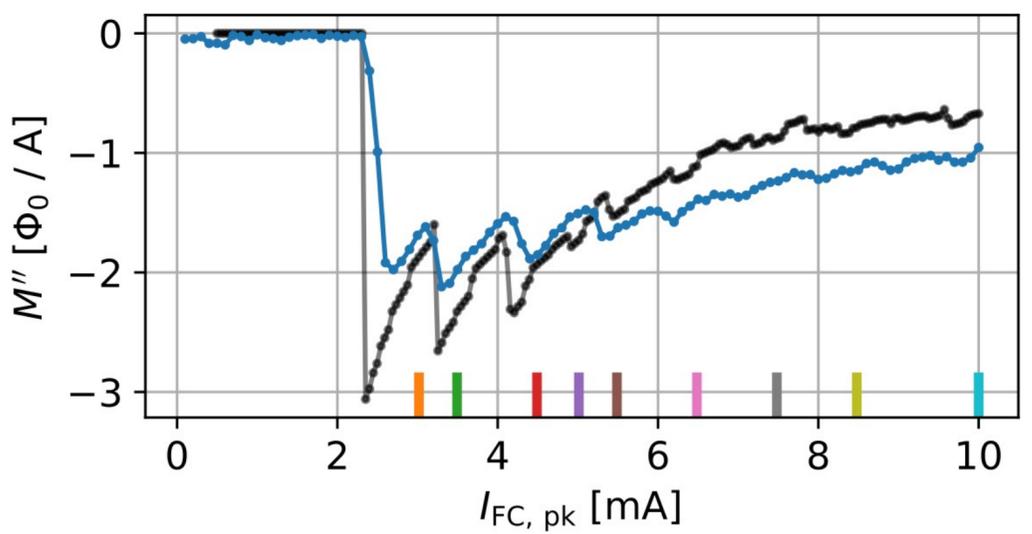
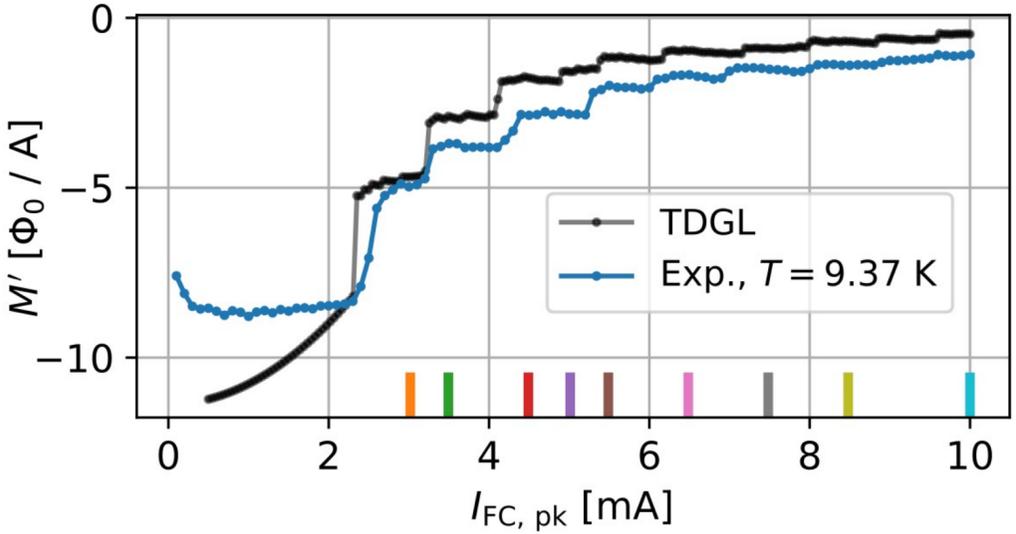


Non-linear, hysteretic response. Many V-aV pairs induced each half AC cycle; some aV pinned far from SQUID.



Simulation captures the observed demodulated magnetic response

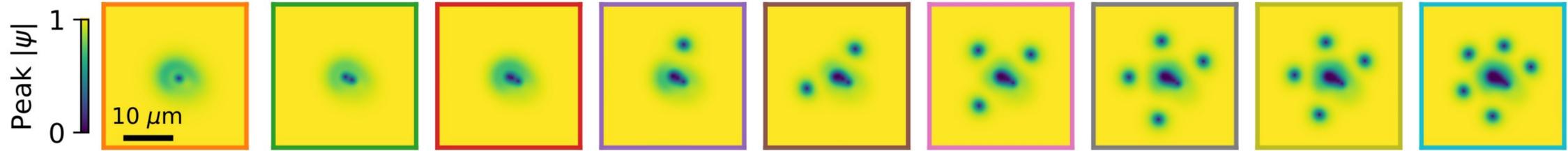
Demodulated mutual inductance signal



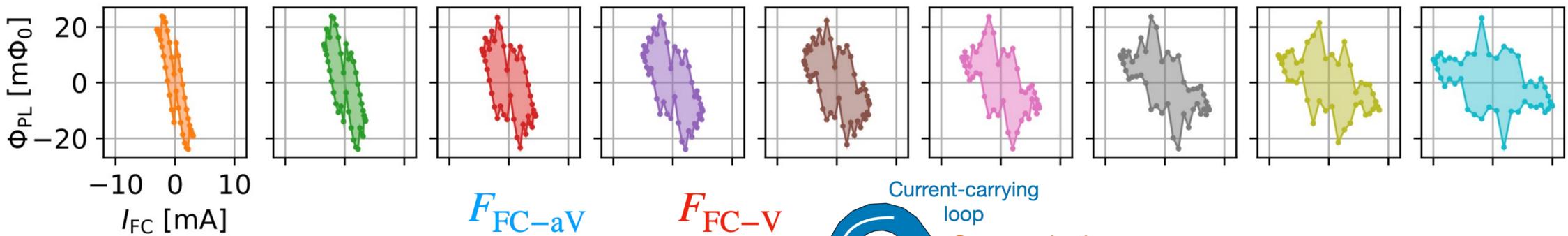
$\lambda = 1.35 \mu\text{m}$
 $\xi = 0.9 \mu\text{m}$

Vortex configurations

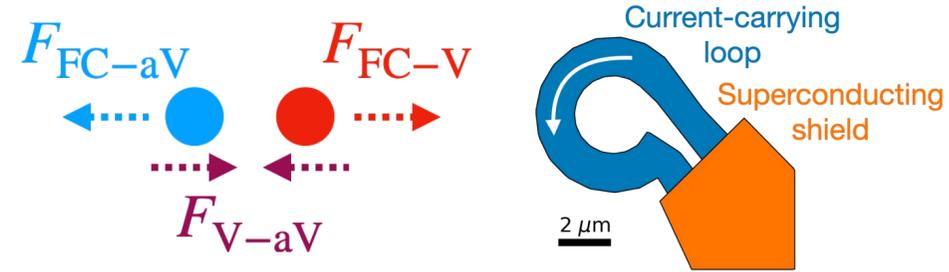
$I_{FC, pk} = 3.0 \text{ mA}$ ($N_V, N_{aV} = (1, 0)$) 3.5 mA (2, 0) 4.5 mA (3, 0) 5.0 mA (4, 1) 5.5 mA (5, 2) 6.5 mA (6, 3) 7.5 mA (7, 4) 8.5 mA (8, 4) 10.0 mA (10, 5)

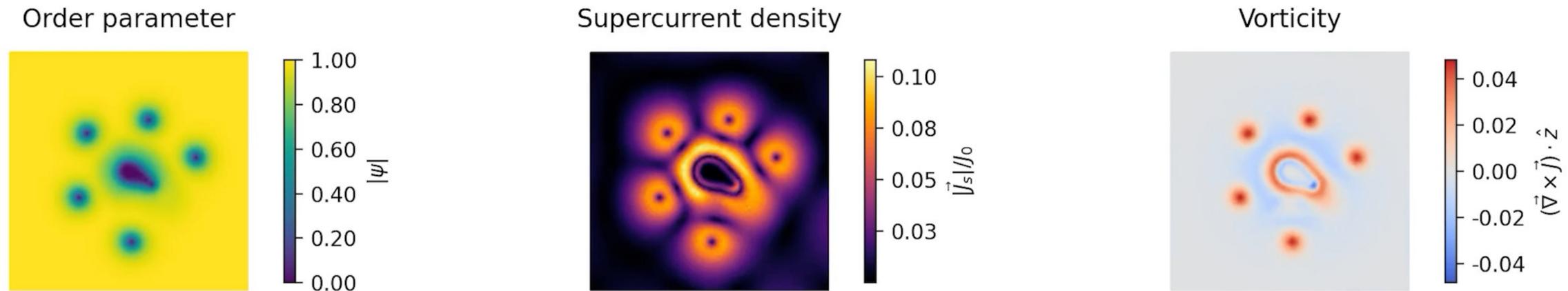


Hysteresis loops



Area of hysteresis loop \propto energy dissipated per cycle

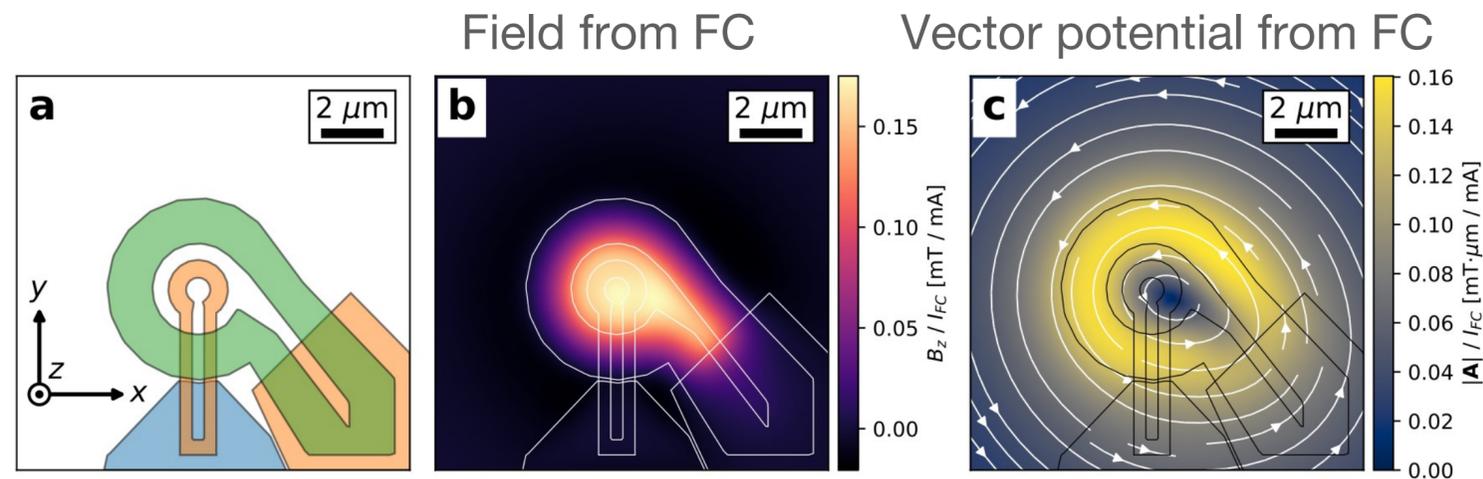




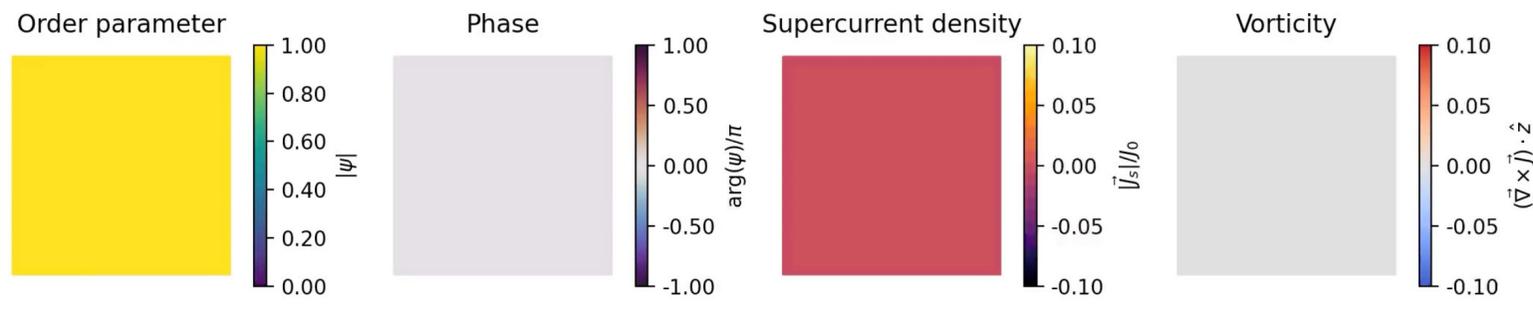
- Scanning SQUID susceptometry + TDGL modeling enables local studies of vortex dynamics resolved at the single-vortex level
- Local approach is complementary to global AC susceptibility and transport, which can be dominated by the surface barrier
- Can be easily extended to study inhomogeneous 2D superconductors, engineered pinning, etc.

Ingredients for modeling vortex dynamics induced by SQUID susceptometry

1. Model field applied to sample by SQUID for a given I_{FC}
2. Model sample response to applied field
3. Calculate the flux that the SQUID sees, Φ_{PL} , due to \mathbf{J}_S in the sample
4. Demodulate $\Phi_{PL}(t)$ over a full AC cycle to get $M = M' + iM''$



Comp. Phys. Comm. **280**, 108464 (2022), superscreen.readthedocs.io

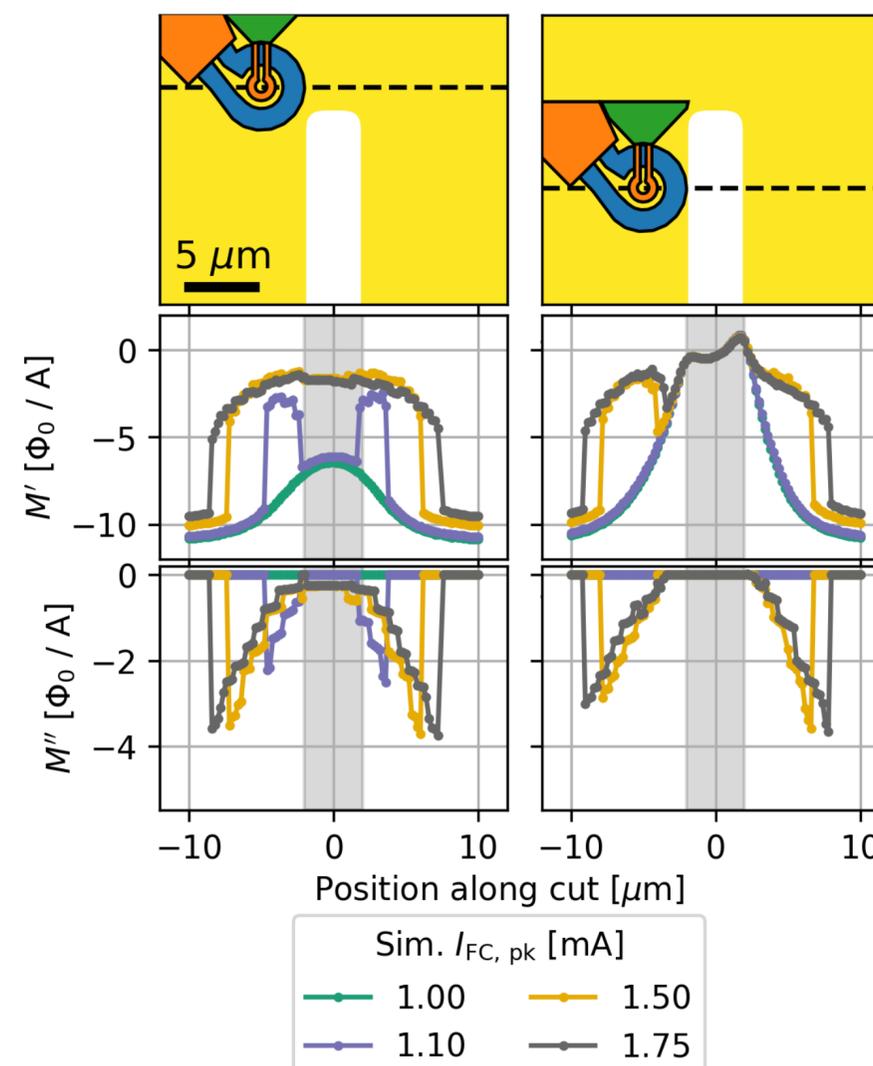
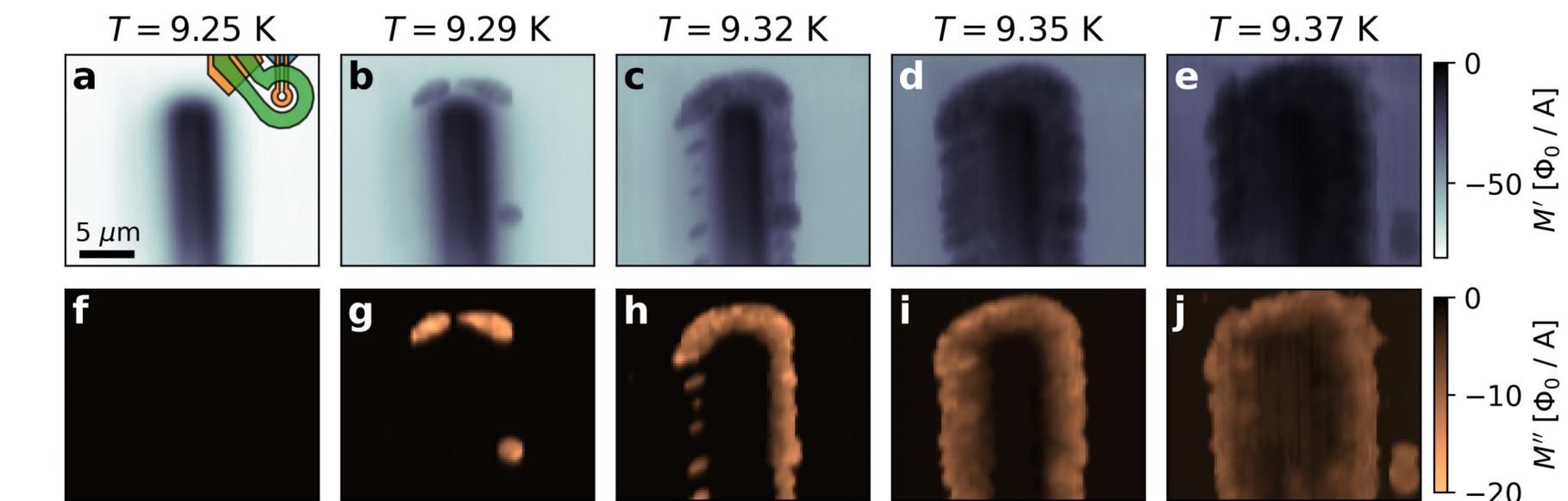
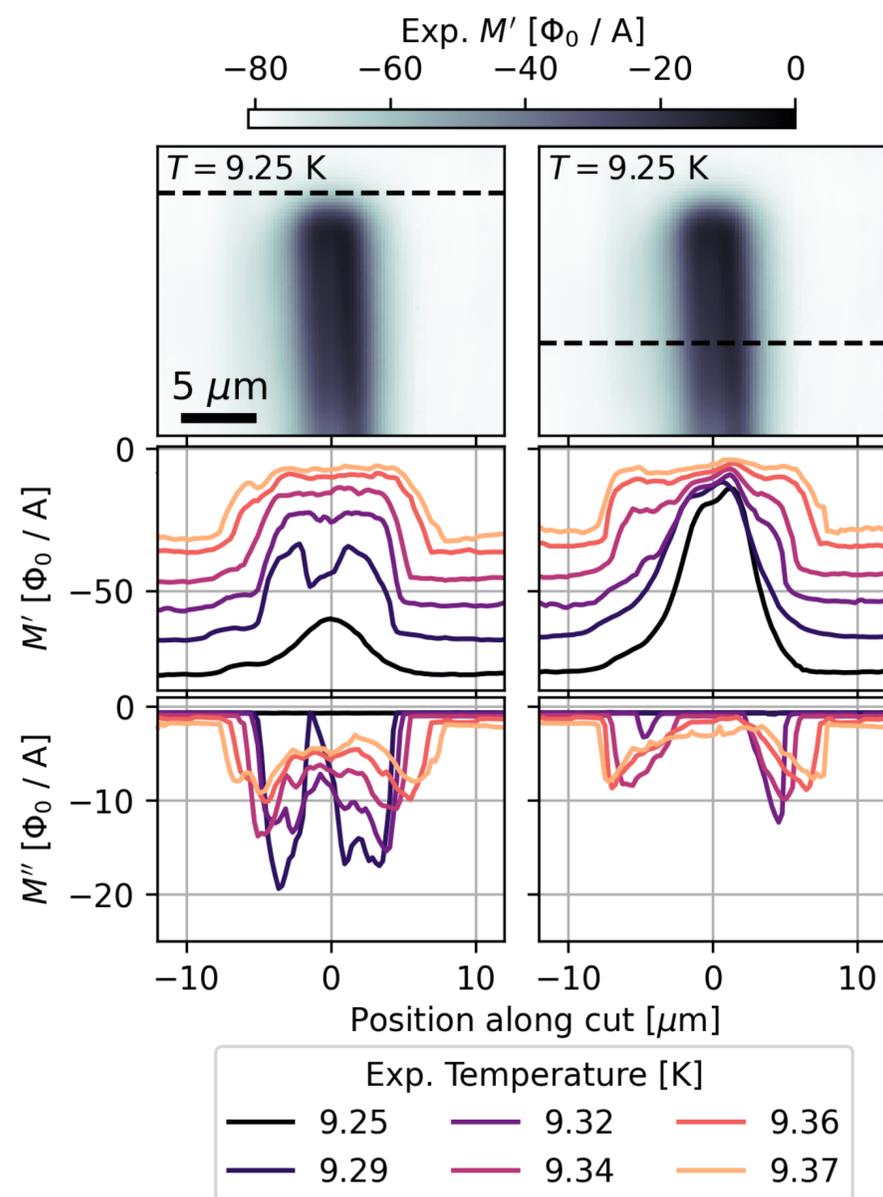


arXiv:2302.03812 (2023); py-tdgl.readthedocs.io

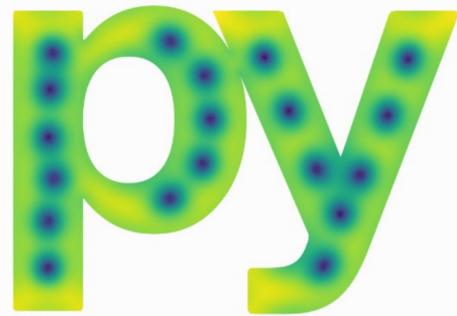
Demodulated magnetic response

$$M = \frac{\sqrt{2}}{I_{FC,pk}} \int \Phi_{PL}(t) e^{-i\omega t} dt = M' + iM''$$

Magnetic response near a lithographically defined defect



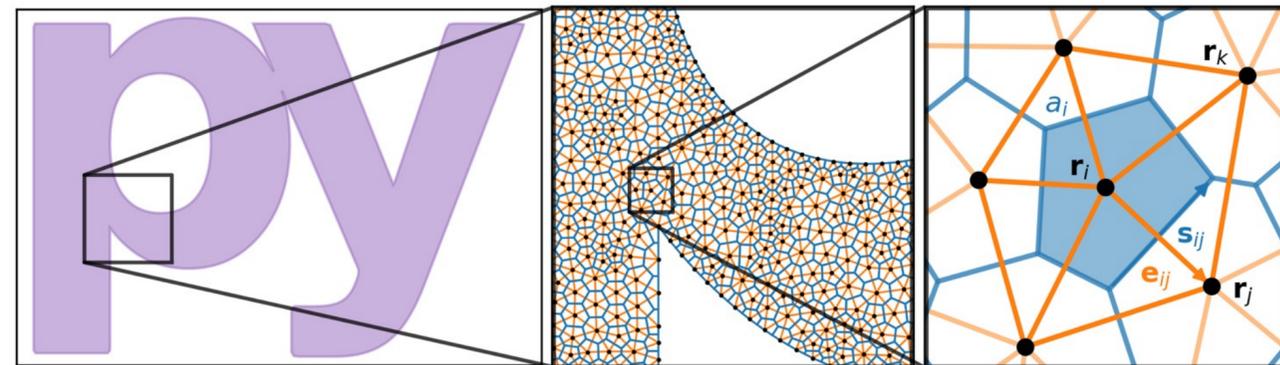
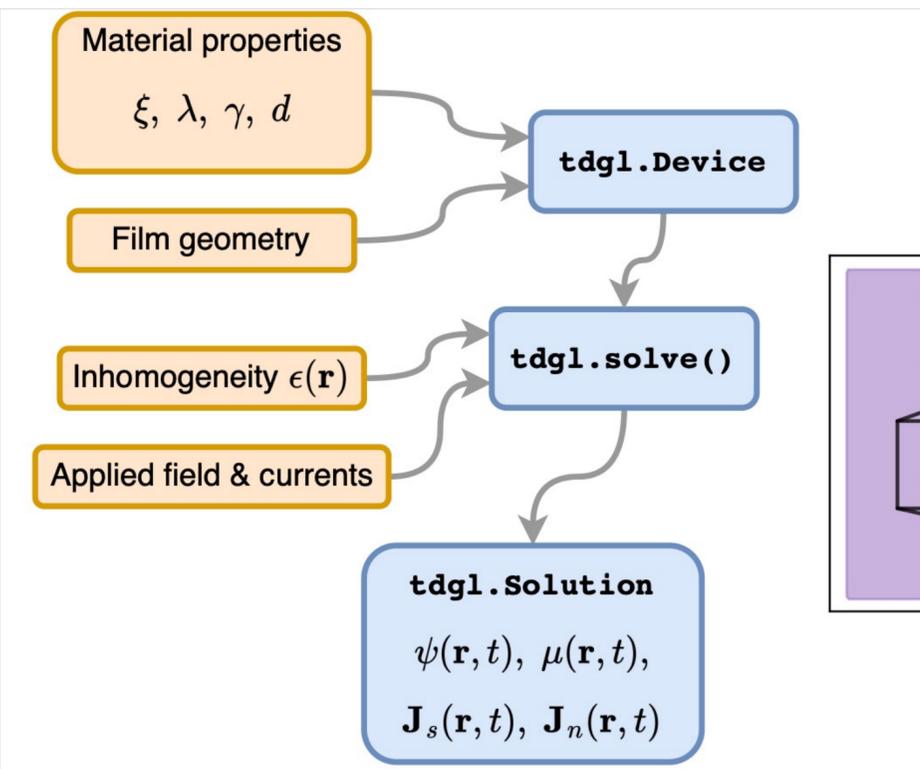
pyTDGL: Time-dependent Ginzburg Landau in Python



pypi v0.2.1
build passing
docs passing
codecov 91%
license MIT
code style black

DOI 10.5281/zenodo.7618841

arXiv:2302.03812 (2023); py-tdgl.readthedocs.io



$$\frac{u}{\sqrt{1 + \gamma^2 |\psi|^2}} \left(\frac{\partial}{\partial t} + i\mu + \frac{\gamma^2}{2} \frac{\partial |\psi|^2}{\partial t} \right) \psi = (\epsilon - |\psi|^2) \psi + (\nabla - i\mathbf{A})^2 \psi$$

$$\nabla^2 \mu = \nabla \cdot \text{Im}[\psi^* (\nabla - i\mathbf{A}) \psi] = \nabla \cdot \mathbf{J}_s$$

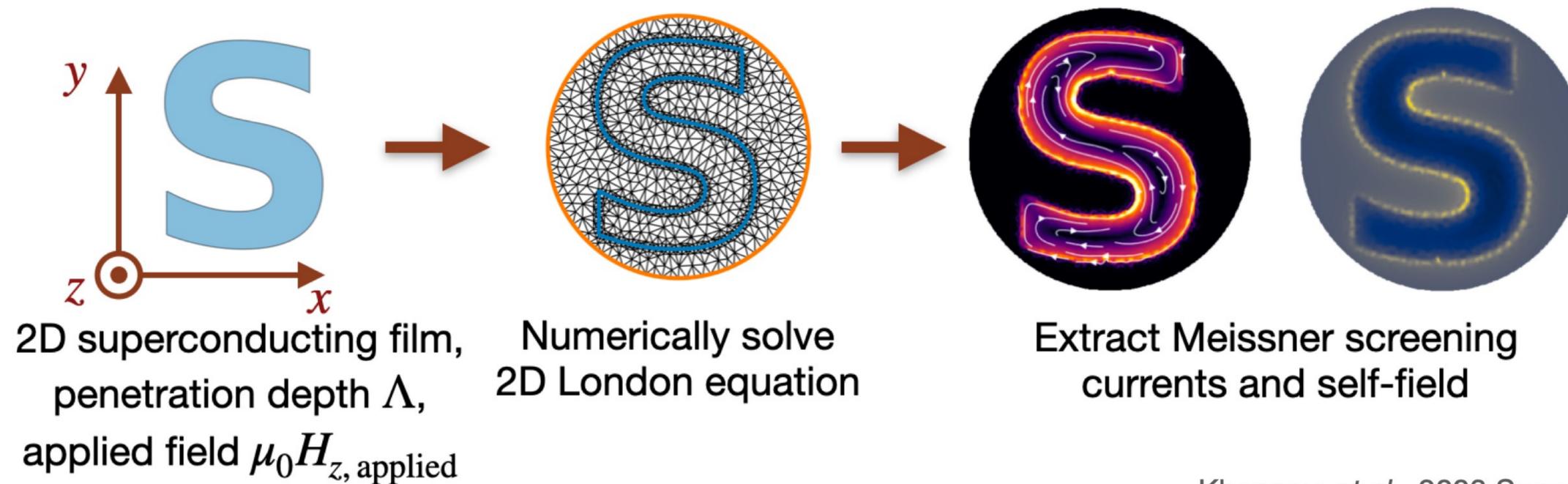
$\psi(\mathbf{r}, t)$: Complex order parameter
 $\mu(\mathbf{r}, t)$: Electric scalar potential
 $\mathbf{J}_s(\mathbf{r}, t)$: Supercurrent density
 $\mathbf{J}_n(\mathbf{r}, t) = \nabla \mu$: Normal current density
 $\epsilon(\mathbf{r}) = T_c(\mathbf{r})/T - 1$: Local critical temperature
 $\gamma \propto \tau_E \Delta_0$: Inelastic scattering (vortex viscosity)

Kramer, *et al.*, Phys. Rev. Lett. **40** (15) (1978)
 Watts-Tobin, *et al.*, J. Low Temp. Phys. **42** (5) (1981)
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 Du, *et al.*, SIAM Journal on Numerical Analysis, **35**, 3 (1998)
 M. Jonsson, Ph.D. thesis, KTH (2022)
 LBVH, arXiv:2302.03812 (2023)

SuperScreen: An open-source package for simulating the magnetic response of two-dimensional superconducting devices ☆, ☆☆

Logan Bishop-Van Horn^{a,c}  , Kathryn A. Moler^{a,b,c} 

arXiv:2203.13388 (2022), superscreen.readthedocs.io



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Kirtley, *et al.*, Supercond. Sci. Technol. **29** 124001 (2016)
Kirtley, *et al.*, Rev. Sci. Instrum. **87**, 093702 (2016)
LBVH & K. Moler, Comp. Phys. Comm. **280**, 108464 (2022)